OBJECTIVES (MCQ'S) OF CHAPTER-1 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Functions Domain & Range:

1. A function $f:X \to Y$ defined by $f(X)=a \forall x \varepsilon X$ and $a \varepsilon Y$ is called **(3 times)**
(A) Linear function (B) Identity function
(C) Constant function (D) Non-linear function

2. The area of a circle as a function of its circumference 'C' is : **(2 times)**
(A) $A=\frac{1}{2\pi}C$ (B) $A=\frac{1}{4\pi}C$ (C) $A=\frac{1}{4\pi}C^2$ (D) $\frac{1}{\pi}C^2$

3. $\cos h^2 x + sinh^2 x =$ **(2 times)**
(A) 1 (B) cosh 2x (C) sinh 2x (D) 2 cosh 2x

4. If f(x) is a function such that f(-x) = f(x) then f(x) is said to be : **(3 times)**
(A) Odd function (B) Even function (C) Constant function (D)Linear function

5. A function defined by f(x) = x^3 is : **(5 times)**
(A) Even function (B) Identity function (C) Odd function (D) Linear function

6. $x=at^2$, $y=2at$ are parametric equations of : **(5 times)**
(A) Circle (B) Parabola (C) Hyperbola (D) Ellipse

7. Domain of f(x) = $2+\sqrt{x-1}$ is : **(3 times)**
(A) (0, 1] (B) $[2,\infty)$ (C) $[1,\infty)$ (D) $[0,\infty)$

8. $x=a\cos\theta$, $y=b\sin\theta$ are parametric equations of : **(3 times)**
(A) Parabola (B) Ellipse (C) Circle (D) Hyperbola

9. A function defined by f(x) = x^2 is : **(4 times)**
(A) Odd function (B) Linear function (C) Even function (D) Constant function

10. If f(x) = cos x, then f(0) = ?: **(2 times)**
(A) -1 (B) -1/2 (C) 0 (D) 1

11. 2sinhx is equal to: **(5 times)**
(A) e^x+e^{-x} (B) e^x-e^{-x} (C) $\frac{e^x-e^{-x}}{e^x+e^{-x}}$ (D) $\frac{e^x+e^{-x}}{e^x-e^{-x}}$

12. f(x) = cos x + sin x is ______ function: **(3 times)**
(A) Even (B) Odd (C) Both even & odd (D) Neither even nor odd

13. The function is said to be an even function if f(-x) = ______: **(1 time)**
(A) –f (x) (B) f(-x) (C) f(x) (D) None of these

14. If y is image of x under function f we write it as: **(3 times)**
(A) x = f(y) (B) y ≠ f(x) (C) y = f (x) (D) y = x

15. If f(x) = 2x + 5 then f(2) equals: **(3 times)**
(A) 1 (B) 9 (C) – 9 (D) – 1

16. Which of the following is an odd function? **(6 times)**
(A) cos x (B) coshx (C) sinhx (D) $sin^2 x$

17. If f(x) = $\sqrt{x+1}$ domain of f equal to: **(5 times)**
(A) (1, + ∞) (B) (-1, - ∞) (C) [-1, +∞) (D) (-∞, +∞)

18. Cosh x is equal to.
(A) $\frac{e^x+e^{-x}}{2}$ (B) $\frac{e^x-e^{-x}}{2}$ (C) e^x (D) $e^x + e^{-x}$

19. **If f (x) = x sec x then f (0)**

(A) – 1 (B) 2 (C) 0 (D) ∞

20. **The expression** $\ell n(x+\sqrt{x^2+1})$ **equals:**

(A) $\sinh^{-1} x$ (B) $\cosh^{-1} x$ (C) $\tanh^{-1} x$ (D) cosech⁻¹x

21. $\cosh^2 x - \sinh^2 x =$

(A) -2 (B) -1 (C) 1 (D) 2

22. $sech^2 x =$

(A) $cosh^2 x =$ (B) $1+\tanh^2 x =$ (C) $1-\tanh^2 x$ (D) $\tanh^2 x - 1$

23. **If f(x) = $x^{2/3}$ + 6 then f (0) =**

(A) 1 (B) 4 (C) 6 (D) 8

24. $f(x)$ **= x cot x is:**

(A) Linear function (B) Quadratic function (C) Even function (D) Odd function

25. **f(x) =** $\frac{3x}{x^2+1}$ **is __________ function.**

(A) Even (B) Odd
(C) Both even and odd (D) Neither even nor odd

26. **The output of a function is also called:**

(A) Result (B) Domain (C) Image (D) None of these

27. $\frac{1-e^{2x}}{2e^x}$ **is equal to**

(A) sinh x (B) –sinh x (C) cosh x (D) –cosh x

28. f**(x) = x + 1 is a function then** f **(t^2-1) = __________** **(2 times)**

(A) t+1 (B) t^2+1 (C) t (D) t^2

29. **If** f**(-x) = –**f **(x) then** f **is called**

(A) Linear function (B) Periodic function (C) Odd function (D) Even function

30. **Who recognized the term function to describe the dependence of one quantity on other.**

(A) Euler (B) Leibniz (C) Langrange (D) Bohr

31. **If f (x) = x^2 , then domain of f is:** **(2 times)**

(A) real No. (B) Integer (C) rational No. (D) irrational

32. **If f(x) = x^2 – x then f (-2) is equal to:**

(A) 2 (B) 6 (C) 0 (D) -6

33. **The range of f(x) = x^2 is** **(2 times)**

(A) (-∞,∞) (B) (-∞,0) (C) [0,∞) (D) (-1,0)

34. **If (x) = x^3 + sinx then f(x) is**

(A) constant function (B) Even function (C) Odd function (D) Neither even nor odd

Topic II: Composition of Functions and Inverse of Function:

35. **If** $f(x) = -2x+8$ **then** $f^{-1}(x) =$: **(5 times)**

(A) $\frac{8+x}{2}$ (B) $\frac{x-8}{2}$ (C) $\frac{8-x}{2}$ (D) $\frac{2}{8-x}$

36. **If f(x) = 2x + 1, then fof(x) is equal to :** **(6 times)**

(A) 4x + 1 (B) 4x + 3 (C) 4x – 3 (D) 4x – 1

37. **If f(x) = $(-x + 9)^3$, then $f^{-1}(x)$ equals.**

(a) $x^{1/3} - 9$ (b) $(x-9)^{1/3}$ (c) $9 - x^{1/3}$ (d) $9 + x^{1/3}$

38. **If g(x) =** $\frac{1}{x^2}$ **(x ≠ 0) then gog(x)** **(2 times)**

(a) 1 (b) x^2 (c) x^4 (d) $\frac{1}{x^4}$

39. If $f(x) = 2x - 1$, then $f^{-1}(x)$ equals **(2 times)**

(A) 1-x (B) 1+x (C) $\frac{1-x}{2}$ (D) $\frac{1+x}{2}$

Topic III: Limit of Function:

40. $\lim\limits_{x\to 0}\frac{a^x-1}{x}$ is equal to : **(4 times)**

(A) $\ell n\, x$ (B) $\log_e a$ (C) 1 (D) 0

41. $\lim\limits_{x\to 0}\frac{x^2}{\sin 7x\sin 5x}$; **(2 times)**

(A) $\frac{7}{5}$ (B) $\frac{5}{7}$ (C) 2 (D) $\frac{1}{35}$

42. $\lim\limits_{x\to 0}\left(\frac{1}{e^{-x}}\right)$ equals : **(1 time)**

(A) 0 (B) 1 (C) -1 (D) $-\infty$

43. $\lim\limits_{x\to 2}\frac{x-2}{\sqrt{x}-\sqrt{2}} =$ ______ **(2 times)**

(A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) 0

44. $\lim\limits_{x\to 1}\frac{x^3-3x^2+3x-1}{x^3-x}$ equals: **(3 times)**

(A) 0 (B) -1 (C) 1 (D) 3

45. $\lim\limits_{x\to 0}\frac{e^x-1}{x} =$ ______ : **(4 times)**

(A) 0 (B) 1 (C) e (D) ∞

46. $\lim\limits_{x\to 0} e^{1/x} =$ ______ , $x<0$ **(2 times)**

(A) -1 (B) 0 (C) 1 (D) ∞

47. $\lim\limits_{\theta\to 0}\frac{1-\cos\theta}{\theta}$ equals: **(4 times)**

(A) 0 (B) -1 (C) 1 (D) 2

48. $\lim\limits_{x\to 0}\frac{\sin x^0}{x} =$ **(2 times)**

(a) 1 (b) 0 (c) $\frac{\pi}{180}$ (d) $\frac{180}{\pi}$

49. $\lim\limits_{n\to\infty}(1+\frac{1}{n})^{2n} =$

(a) e^2 (b) e^{-1} (c) $e^{-/2}$ (d) e

50. $\lim\limits_{n\to\infty}(1+\frac{1}{n})^{n} =$ **(2 times)**

(a) 4e (b) 3e (c) 2e (d) e

51. $\lim\limits_{x\to 0}\frac{\sin ax}{\sin bx} =$

(A) $\frac{a}{b}$ (B) $\frac{-b}{a}$ (C) $\frac{-a}{b}$ (D) $\frac{b}{a}$

52. $\lim\limits_{n\to\infty} it \left(1+\frac{1}{3n}\right)^n =$

(A) e^3 (B) $e^{1/2}$ (C) $e^{\frac{1}{3}}$ (D) $-{}^{1}/{}_{e^3}$

53. $\lim_{x \to a} \frac{x^2-a^2}{x-a} =$:

(A) 0 (B) 2a (C) a^2 (D) Unde.

54. $\lim_{x\to 0}(1+3x)^{2/x}$

(A) e^2 (B) e^8 (C) e^6 (D) e^4

55. $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ is equal to:

(A) f′ (x) (B) f′ (a) (C) f′(2) (D) f′(0)

56. If $\lim_{x \to 0} \frac{f(x)-1}{x} = \ell na$; a>0, then f(x) is:

(A) a^{-x} (B) a^x (C) e^{-x} (D) e^x

57. $\lim_{x \to \infty}\left(1+\frac{x}{2}\right)^{\frac{1}{x}}$ equals :

(A) e (B) e^{-1} (C) e^2 (D) $\sqrt{e}$

Topic IV: Continuous and Discontinuous Function:

58. The function $f(x) = \frac{x^2-1}{x-1}$ is discontinuous at : **(3 times)**

(A) 0 (B) 1 (C) -1 (D) 2

59. The function $f(x) = \frac{2+3x}{2x}$ is not continuous at.

(a) x = - 3 (b) $x = -\frac{2}{3}$ (c) x = 0 (d) x = 1

60- If $f(x)=\sqrt{x+4}$ then $f=(x^2+4)$ is equal to:

(A) x^2-8 (B) $\sqrt{x^2-8}$ (C) $\sqrt{x^2+8}$ (D) x^2+8

61- $\lim_{x\to 0} \frac{\sin 7x}{x}$ is equal to:

(A) 1 (B) 7 (C) $\frac{1}{7}$ (D) 0

62- $x = a\cos\theta$, $y = b\sin\theta$ represent:

(A) Circle (B) Parabola (C) Ellipse (D) Hyperbola

63- $Log_e\left(\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right) = ----, \quad 0<x\le 1$

(A) $\text{Sech}^{-1}x$ (B) $\text{Cosech}^{-1}x$ (C) $\text{Tanh}^{-1}x$ (D) $\text{Coth}^{-1}x$

64- The linear function f(x) = ax + b becomes identity function if:

(A) a = 0, b = 1 (B) a = 1, b = 0 (C) a = 0, b = 0 (D) a = 1, b = 1

65- If $f(x)=\sqrt{x+4}$, then f(4) =:

(A) 8 (B) 16 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

66- If f(x) = -2x + 6, then $f^{-1}(x)$ =:

(A) 6 – 2x (B) $\frac{6-x}{2}$ (C) $\frac{2}{6-x}$ (D) 2x – 6

67- Let f(x) = x^2 + cosx, then f(x) is:

(A) Odd function (B) Constant function (C) Even function (D) Neither even nor odd

68- $\lim_{n\to +\infty}\left(1+\frac{3}{n}\right)^{2n}$ =:

(A) e (B) e^2 (C) e^3 (D) e^6

69- The parametric equations x = a sec θ and y = b tan θ represent the equation of:

(A) Hyperbola (B) Circle (C) Parabola (D) Ellipse

70. Domain of f (x) = x^2 + 1 is:

(a) R (b) R – {1} (c) R – {-1} (d) $[1,\infty)$

71. $\frac{e^x - e^{-x}}{2}$ = ________

(a) sin x (b) cos x (c) sin h x (d) cos h x

72. $\lim_{x\to 4}\frac{x^2-6x+8}{x-4}=$

(a) 4 (b) 2 (c) 6 (d) 8

73. $\lim_{\theta\to 0}\frac{1-\cos p\theta}{1+\cos p\theta}$ **equals:**

(a) 1 (b) 0 (c) $\frac{p^2}{q^2}$ (d) 2

74. The range of $f(x)=x^2$ **is:**

(a) $(-\infty,0)$ (b) $(-\infty,\infty)$ (c) $(-1,0)$ (d) $[0,\infty)$

75. If $f(x)=\frac{1}{x^2}(x\neq 0)$, **then** $fof(x)$ **is**

(a) x^4 (b) x^2 (c) 1 (d) $\frac{1}{x^4}$

ANSWERS TO THE MULTIPLE CHOICE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	c	b	b	c	b	b	b	c	d	b	d	c	c	b
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
c	c	a	c	a	c	c	c	c	b	c	b	d	c	b
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
a	b	c	c	c	b	c	c	d	b	d	b	c	a	b
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
b	a	c	a	d	a	c	b	c	b	b	d	b	c	c
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
b	c	a	b	d	b	c	d	a	a	c	b	b	d	a

SHORT QUESTION'S OF CHAPTER-1
ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Functions Domain & Range:

1. If $f(x)=\sqrt{x+4}$ **then find** $f(x^2+4)$ **(C.W)**

Sol : $f(x)=\sqrt{x+4}$

$f(x^2+4)=\sqrt{x^2+4+4}$

$f(x^2+4)=\sqrt{x^2+8}$ **Which is required.**

2. **Find the domain and range of $\sqrt{x^2-4}$** (C.W) (2 times)

Sol : Domain of f = R − (-2, 2)

Range of f = [0, ∞)

3. **Define an implicit function.** (4 times)

Sol : If x and y be so mixed that y can't express in term of x then y is called on implicit function. Example $y^2 = x^3y - x^2y^2 + 5$

4. **Let $f(x) = \sqrt{x^2-9}$, find the domain and range of f.**

Sol : $f(x) = \sqrt{x^2-9}$

Domain f = R − (-3, 3)

Range f = $[0,\infty)$

5. **Determine whether f(x) = $x^{2/3}+6$ is even or odd.** (C.W) (2 times)

Sol : f(x) = $x^{2/3}+6$

Now f(-x) = $(-x)^{2/3}+6 = \left[(-x)^2\right]^{1/3}+6$

$= \left[x^2\right]^{1/3}+6 = x^{2/3}+6 =$ f(x)

So f(x) is an even function.

6: **If $f(x) = \sin x + \cos x$. Check whether f is even or odd.** (C.W)

Sol: Let f(x) = Sinx + Cosx

Put x = -x in eq. (1)

⇒f(-x) = Sin (-x) + Cos (-x)

⇒f(-x) = -Sinx + Cosx

$\Rightarrow f(-x) \neq \pm f(x)$

Hence f(x) is neither even nor odd.

7: **Define Even and odd functions.** (6 times)

Sol: **Even function:** A function f is said to be an even if f(-x) = f(x)

i.e. f(x) = x^2

f(-x) = $(-x)^2 = x^2$ = f(x)

Odd function: A function f is said to be odd function if f(-x) =-f(x)

$f(x) = x^3$ *replace x by −x*

$f(-x) = (-x)^3 = -x^3 = -f(x)$

Hence f(x)be an odd function.

8: **Find the Domain and Range of : g(x)** $= \begin{cases} 6x+7, & \text{if } x \le -2 \\ 4x-3, & \text{if } -2 < x \end{cases}$ (C.W)

Sol: g(x) $= \begin{cases} 6x+7, & \text{if } x \le -2 \\ 4x-3, & \text{if } -2 < x \end{cases}$

We see that g(x) is defined for all real values of x.so

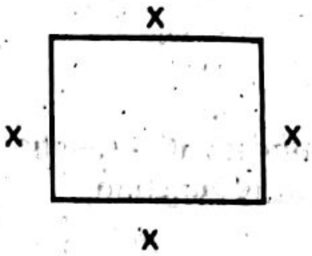
Domain g = Set of all real numbers = $(-\infty, \infty)$

Range g = Set of all real numbers = $(-\infty, \infty)$

9: **Express the perimeter P of square as a function of its area A.**

(H.W) (3 times)

Sol: Consider a square with x as length of each side If P is perimeter and A as Area then



$P = x + x + x + x$
$P = 4x$
and $A = x^2$
Now $P = 4x$
$= 4\sqrt{x^2} = 4\sqrt{A}$
So $P = 4\sqrt{A}$

10: **$f(x) = x^3 - ax^2 + bx + 1$, If $f(2) = -3$ and $f(-1) = 0$ Find values of a and b (H.W)**

Sol: Given function iscf(x) $= x^3 - ax^2 + bx + 1$
As $f(2) = -3$ So
$(2)^3 - a(2)^2 + 2b + 1 = -3$
$8 - 4a + 2b + 1 = -3$
$-4a + 2b = -12$
$2a - b = 6$ (1)
As $f(-1) = 0$
$(-1)^3 - a(-1)^2 + b(-1) + 1 = 0$
$-1 - a - b + 1 = 0$
$a + b = 0$ (2)
Adding (1) and (2)
$3a = 6 \quad \Rightarrow a = 2$
Put in equation (2)
$a + b = 0$
$2 + b = 0$
$b = -2$

11: **If $f(x) = \sin x$, then find $\frac{f(a+b) - f(a)}{h}$ (H.W)**

Sol: Given $f(x) = \sin x$
$\Rightarrow f(a + b) = \sin(a + b)$
$f(a) = \sin a$
$\frac{f(a+h) - f(a)}{h} = \frac{\sin(a+b) - \sin a}{h}$
$= \frac{2\cos\left(\frac{a+h+a}{2}\right)\sin\left(\frac{a+h+a}{2}\right)}{h} = \frac{2}{h}\cos\left(a + \frac{h}{2}\right)\sin\frac{h}{2}$

12. **Show that $x = at^2$, are parametric equations of parabola $y = 2at$ (C.W)**

Sol **Given parametric equation of parabola are.**
$x = at^2$ ______(1) $y = 2at$ ________(2)
squaring eq (2)
$y^2 = 4a^2t^2$
$y^2 = 4a(at^2)$ ________(3)
Put eq (1) in (3)
$y^2 = 4ax$
Which is required equation of parabola

13. **Evaluate** $\lim_{h \to 0} (1 + 2h)^{1/h}$ **(C.W)**

Sol **Given** $\lim_{h \to 0} (1 + 2h)^{1/h}$
$= \lim_{h \to 0} (1 + 2h)^{2/2h}$

$$= \left(\lim_{h \to 0} (1+2h)^{2/2h} \right)^2 \qquad \because \lim_{x \to 0} (1+x)^{1/x} = e$$

$$= (e)^2$$

$$= e^2$$

14. Define identity function.

Sol **Identity Function:-**

Let a function $I : X \to X$ of the form $I(x) = x, \ \forall x \in X$ Is called an identity function. Ans.

15. $f(x) = x^2 - x$ then find $f(x-1)$. **(C.W)**

Sol Given $f(x) = x^2 - x$ Find $f(x-1)$

Now $f(x-1) = (x-1)^2 - (x-1)$

$f(x-1) = x^2 - 2x + 1 - x + 1$

$f(x-1) = x^3 - 3x + 2$

16. Determine whether the given function is Odd or Even $f(x) = \dfrac{3x}{x^2+1}$ **(C.W)**

Sol Given $f(x) = \dfrac{3x}{x^2+1}$ **replace x by -x**

Now $f(-x) = \dfrac{3(-x)}{(-x)^2+1}$

$$\Rightarrow f(-x) = \frac{-3x}{x^2+1}$$

$$\Rightarrow f(-x) = -\left(\frac{3x}{x^2+1}\right)$$

$$\Rightarrow f(-x) = -f(x)$$

Hence f(x) be an odd function.

Topic II: Composition of Functions and Inverse of Function:

17. Without finding inverse state Domain and Range of $f^{-1}(x)$ where $f(x) = 2+\sqrt{x-1}$ **(C.W)**

Sol: We see f is not defined when $x < 1$

Domain $f = [1, +\infty)$

Range $f = [2, +\infty)$

So

Domain $f^{-1} = \text{Range } f = [2, +\infty)$

Range $f^{-1} = \text{Domain } f = [1, +\infty)$

18. If $f(x) = \sqrt{x+1}$; $g(x) = \dfrac{1}{x^2}$ find fog (x) **(H.W)**

Sol: fog(x)

$f(x) = \sqrt{x+1}, \ g(x) = \dfrac{1}{x^2}$

fog (x) = f [g(x)]

fog (x) = $f\left[\dfrac{1}{x^2}\right]$

fog (x) = $\sqrt{\dfrac{1}{x^2}+1} = \sqrt{\dfrac{1+x^2}{x^2}} = \dfrac{\sqrt{x^2+1}}{x}$

19: Verify $f(f^{-1}(x)) = x$, where $f(x) = (-x+9)^3$ (H.W) (2 times)

Sol: Given $f(x) = (-x+9)^3$

$y = (-x+9)^3$ $\quad \because y = f(x)$

$y^{1/3} = -x+9$

$x = 9 - y^{1/3}$ $\quad \because x = f^{-1}(y)$

$f^{-1}(y) = 9 - y^{1/3}$

Replace y by x.

$f^{-1}(x) = 9 - x^{1/3}$

Now $f(f^{-1}(x))$, $f(9 - x^{1/3})$

$= [-(9 - x^{1/3}) + 9]^3$

$= [-(9 - x^{1/3}) + 9]^3 = (x^{1/3})^3 = x$

$= f(f^{-1}(x)) = x$

20. If $f(x) = \sqrt{x+1}$ *and* $g(x) = \frac{1}{x^2}$ **find gof(x)** (C.W)

Sol Given $f(x) = \sqrt{x+1}$, $g(x) = \frac{1}{x^2}$

Now $gof(x) = g[f(x)] = g(\sqrt{x+1})$

$$= \frac{1}{(\sqrt{x+1})^2} = \frac{1}{x+1}$$

21. If $f(x) = (-x+9)^3$ **find $f^{-1}(x)$** (H.W)

Sol Given $f(x) = (-x+9)^3$

As $y = f(x)$

$y = (-x+9)^3$

$y^{1/3} = [(-x+9)^3]^{1/3}$

$y^{1/3} = -x+9$

$x = 9 - y^{1/3}$

As $x = f^{-1}(y)$

$f^{-1}(y) = 9 - y^{1/3}$

Replacing y by x $\qquad f^{-1}(x) = 9 - x^{1/3}$

Topic III: Limit of Function:

22. **Evaluate** $\lim\limits_{x\to 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$ (C.W) (2 times)

Sol : $\lim\limits_{x\to 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$

$$= \lim_{x\to 3} \frac{(\sqrt{x})^2 - (\sqrt{3})^2}{\sqrt{x}-\sqrt{3}}$$

$$= \lim_{x\to 3} \frac{(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})}{\sqrt{x}-\sqrt{3}}$$

$$= \lim_{x\to 3} (\sqrt{x}+\sqrt{3}) = \sqrt{3}+\sqrt{3} = 2\sqrt{3}$$

23. Evaluate $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n$ **(C.W)**

Sol : $= \lim_{n\to\infty}\left(1+\left(-\frac{1}{n}\right)\right)^n$

$$= \lim_{n\to\infty}\left(1+\left(-\frac{1}{n}\right)\right)^{(-n)(-1)}$$

$$= \left[\lim_{n\to\infty}\left(1+\left(-\frac{1}{n}\right)^{-n}\right)\right]^{-1}$$

$$= e^{-1} = \frac{1}{e}$$

24. Evaluate $\lim_{x\to 2}\frac{x^3-8}{x^2+x-6}$ **(H.W)**

Sol : $\lim_{x\to 2}\frac{x^3-8}{x^2+x-6}\ \left(\frac{0}{0}\ form\right)$

$$\lim_{x\to 2}\frac{x^3-8}{x^2+x-6} = \lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+3)}$$

$$= \frac{\lim_{x\to 2}(x^2+2x+4)}{\lim_{x\to 2}(x+3)} = \frac{\left(\lim_{x\to 2}x\right)^2+2\lim_{x\to 2}x+4}{\lim_{x\to 2}x+3}$$

$$= \frac{(2)^2+2(2)+4}{2+3} = \frac{12}{5}$$

25. Evaluate $\lim_{x\to\infty}\frac{2-3x}{\sqrt{3+4x^2}}$ **(C.W)**

Sol. Here $\sqrt{x^2}=|x|=-x$ as $x<0$

$\therefore$ Dividing up and down by $-x$, we get

$$\lim_{x\to\infty}\frac{2-3x}{\sqrt{3+4x^2}} = \lim_{x\to\infty}\frac{-2/x+3}{\sqrt{3/x^2+4}} = \frac{0+3}{\sqrt{0+4}} = \frac{3}{2}$$

26. Evaluate $\lim_{x\to -1}\frac{x^3-x}{x+1}$ **(H.W) (3 times)**

Sol: $\frac{x^3-x}{x+1} = \frac{x(x^2-1)}{x+1} = \frac{x(x-1)(x+1)}{x+1} = x(x-1)$

$\lim_{x\to -1} x(x-1)$

$= -1(-1-1)$

$= 2$

27: Evaluate $\lim_{x\to 0}(1+2x^2)^{\frac{1}{x^2}}$ **(H.W)**

Sol: Given $\lim_{x \to 0} (1 + 2x^2)^{\frac{1}{x^2}}$

$= \lim_{x \to 0} \left[(1 + 2x^2)^{\frac{1}{2x^2}}\right]^2$

$= \lim_{x \to 0} \left[(1 + 2x^2)^{\frac{1}{2x^2}}\right]^2$ $\qquad \because \lim_{x \to 0} (1 + x)^{1/x} = e$

$= (e)^2 = e^2$

28: **Evaluate** $\lim_{x \to 0} \frac{\sin ax}{\sin bx}$ **(H.W) (6 time)**

Sol: $= \lim_{x \to 0} \frac{\sin ax}{\sin bx}$

$= \lim_{x \to 0} \left(\frac{\frac{\sin ax}{ax} \times ax}{\frac{\sin bx}{bx} \times bx}\right)$

$= \frac{a}{b}\left[\frac{\lim_{x\to 0} \frac{\sin ax}{ax}}{\lim_{x\to 0} \frac{\sin bx}{bx}}\right]$ $\qquad \because \lim_{x \to 0} \frac{\sin\theta}{\theta} = 1$

$= \frac{a}{b}\left(\frac{1}{1}\right) = \frac{a}{b}$

29: **Evaluate :** $\lim_{x \to 0} \frac{1 - \cos x}{x}$ **(H.W) (3 times)**

Sol: $\lim_{x \to 0} \frac{1-\cos x}{x} = \lim_{x \to 0} \left(\frac{1-\cos x}{x}\right)\left(\frac{1+\cos x}{1+\cos x}\right)$

$= \lim_{x \to 0} \frac{1-\cos^2\theta}{x(1+\cos x)} = \lim_{x \to 0} \frac{\sin^2 x}{x(1+\cos x)}$

$= \lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2 \lim_{x \to 0} \frac{x}{1+\cos x} = (1)^2 (0) = 0.$

30: **Evaluate the Limit =** $\lim_{x\to\infty} \left(1 + \frac{3}{x}\right)^{2x}$ **(C.W)**

Sol: Given

$\lim_{x\to 0} \left(1 + \frac{3}{x}\right)^{2x}$

$= \lim_{x \to \infty} \left(\left(1 + \frac{3}{x}\right)^x\right)^2 = \lim_{x \to \infty} \left[\left(1 + \frac{3}{x}\right)^{x/3}\right]^6$

$= \left[\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{x/3}\right]^6$ $\qquad \because \lim_{x \to \infty} \left(1 + \frac{1}{n}\right)^n = e$

$= e^6$

31: **Evaluate** $\lim_{x \to 0} \frac{1-\cos x}{\sin x}$ **(H.W) (2 times)**

Sol: Let I be the required limit then

$I = \lim_{x \to 0} \frac{1-\cos x}{\sin x} = \lim_{x \to 0} \left(\frac{1-\cos x}{\sin x}\right)\left(\frac{1+\cos x}{1+\cos x}\right)$

$= \lim_{x \to 0} \frac{1-\cos^2 x}{\sin x\,(1+\cos x)} = \lim_{x \to 0} \frac{\sin^2 x}{\sin x\,(1+\cos x)}$

$= \lim_{x \to 0} \frac{\sin x}{1+\cos x} = \frac{\sin 0}{1+\cos 0} = \frac{0}{1+1} = \frac{0}{2} = 0$

32. **Evaluate** $\lim_{x \to -1} \left(\frac{x^3 + x^2}{x^2 - 1}\right)$ **(H.W)**

Sol $\lim_{x \to -1}\left(\frac{x^3+x^2}{x^2-1}\right)$

Using algebraic technique

$= \lim_{x \to -1} \frac{x^2(x+1)}{(x-1)(x+1)}$

$= \lim_{x \to -1} \left(\frac{x^2}{x-1}\right)$

$= \frac{(-1)^2}{-1-1}$

$= \frac{1}{-2}$

$= -\frac{1}{2}$ Ans.

33. Evaluate $\lim_{x \to 0} \frac{1-\cos 2x}{x^2}$ **(H.W)**

Sol **Given** $\lim_{x \to 0} \frac{1-\cos 2x}{x^2}$

$\because 2\sin^2 x = 1-\cos 2x$

$= \lim_{x \to 0} \frac{2\sin^2 x}{x^2}$

$= 2\lim_{x \to 0} \frac{\sin^2 x}{x^2}$

$= 2\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2$

$= 2\left(\lim_{x \to 0} \frac{\sin x}{x}\right)^2$

$\because \lim_{x \to 0} \frac{\sin x}{x} = 1$

$= 2(1)^2$

$= 2$ Ans.

Topic IV: Continuous and Discontinuous Function:

34: Discuss continuity of f(x)= $\begin{cases} \frac{x^2-9}{x-3} &, \text{if } x \neq 3 \\ 6 &, \text{if } x = 3 \end{cases}$ **(C.W) (4 times)**

Sol: Let f(x) be a continuous function at x = 3

(i) f(3) = 6 be defined at x = 3

(ii) $\lim_{x \to 3}$ f(x) be exists.

$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)}$

$= \lim_{x \to 3} (x+3)$

$= 3 + 3 = 6$

(III) $\lim_{x \to 3} f(3) = f(3)$

$6 = 6$

Hence f(x) be a continuous function at x = 3

35: Discuss continuity of f(x) at 3. When f(x) = $\begin{cases} x-1 & , if\ x<3 \\ 2x+1 & , if\ x\geq 3 \end{cases}$ **(H.W) (2 times)**

Sol: Let function be defined at x = 3

(I) $f(3) = 2(3) + 1 = 6 + 1 = 7$

(ii) Let $\lim_{x \to 3} f(x)$ will be exists.

If $\lim_{x \to 3} f(x) = \lim_{x \to 3} + f(x)$

$\lim_{x \to 3} f(x) = \lim_{x \to 3^-} (x-1) = 3 - 1 = 2$

$\lim_{x \to 3} f(x) = \lim_{x \to 3^+} (2x+1) = 2(3) + 1 = 6 + 1 = 7$

Since $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$

Condition (ii) is not satisfied

Hence f(x) is not continues at x = 3

36: Define Continuity.

Sol: A Function f is said t

o be continuous at 'c' iff the following three conditions are satisfied.

(i) f(c) is defined (ii) $\lim_{x \to c} f(x)$ exists (iii) $\lim_{x \to c} f(x) = f(c)$.

37. Discuss continuity of function $f(x) = \begin{cases} 2x+5 & , if\ x \leq 2 \\ 4x+1 & , if\ x > 2 \end{cases}$ **at x=2 (C.W)**

Sol Given

$$f(x) = \begin{cases} 2x+5 & , if\ x \leq 2 \\ 4x+1 & , if\ x > 2 \end{cases}$$

be a continuous function at x = 2

(i) $f(2) = 2(2) + 5 = 9$

(ii) $\lim_{x \to 2} f(x)$ be exists.

So L.H.L = R.H.L

$$\text{L.H.L} = \lim_{x \to 2^-} f(x)$$

$$\text{L.H.L} = \lim_{x \to 2^-} (2x+5)$$

$$= 2(2)+5$$

$$= 4+5$$

$$= 9$$

$$R.H.L = \lim_{x \to 2^+} f(x)$$

$$= \lim_{x \to 2^+} (4x+1)$$

$$= 4(2)+1 \quad = \quad 8+1 \quad = \quad 9$$

L.H.L = R.H.L

So $\lim_{x \to 2} f(x)$ be exists

(iii) $f(2) = \lim_{x \to 2} f(x)$

$9 = 9$

Hence all conditions of continuous function be satisfy. So f(x) be a continuous function

At $x = 2$ Ans.

38. If $f(x) = \begin{cases} x+2 & , \quad x \le -1 \\ c+2 & , \quad x > -1 \end{cases}$**, Find 'c'. So that** $\lim_{x \to -1} f(x)$ **exists. (H.W)**

(2 times)

Sol: Since $\lim_{x \to -1} f(x)$ be exists

So Left Hand limit = Right hand limit

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$$

$$\lim_{x \to -1^-} (x+2) = \lim_{x \to -1^+} (c+2)$$

$$-1+2 = c+2$$

$$1 = c+2$$

$$1-2 = c$$

$$-1 = c$$

$$c = -1$$

39. $f(x) = \dfrac{x}{x^2-4}$, **Find the domain and range of** $f(x)$. **(C.W)**

Sol: At $x = 2$ and $x = -2$, $f(x) = \dfrac{x}{x^2-4}$ is not defined.

So

Domain of $f(x)$ = Set of all real number except -2 and 2

Range of $f(x)$ = Set of all real numbers

40. Find $fog(x)$ **if** $f(x) = \dfrac{1}{\sqrt{x-1}}$, $g(x) = \dfrac{1}{x^2}$, $x \ne 1$ **(H.W)**

Sol: Given $f(x) = \dfrac{1}{\sqrt{x-1}}$, $g(x) = \dfrac{1}{x^2}$

Now $fog(x) = f(g(x))$

$$fog(x) = f\left(\frac{1}{x^2}\right)$$

$$fog(x)=\frac{1}{\sqrt{\frac{1}{x^2}-1}} \qquad =\frac{1}{\sqrt{\frac{1-x^2}{x^2}}}$$

$$fog(x)=\frac{1}{\frac{\sqrt{1-x^2}}{x}} \quad =\frac{x}{\sqrt{1-x^2}}$$ Ans.

41. Find value of 'm' so that f is continuous at x = 3 : $f(x)=\begin{cases} mx & , \text{ if } x<3 \\ x^2 & , \text{ if } x\ge 3 \end{cases}$ **(H.W)**

Sol: Since f(x) be a continuous function so $\lim\limits_{x\to 3} f(x)$ exist.

So $\lim\limits_{x\to 3} f(x)$ be exists

As Left hand limit = Right hand limit

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$$

$$\lim_{x\to 3^-} (mx) = \lim_{x\to 3^+} x^2$$

$$m(3) = (3)^2$$
$$3m = 9$$
$$m = 3 \quad \text{Ans.}$$

42. Determine whether function $f(x)=\frac{x^3-x}{x^2+1}$ is even or odd. **(H.W)**

Sol: Given $f(x)=\frac{x^3-x}{x^2+1}$

Replace x by $-x$

$$f(-x)=\frac{(-x)^3-(-x)}{(-x)^2+1} \quad =\frac{-x^3+x}{x^2+1} \quad =-\left(\frac{x^3-x}{x^2+1}\right)$$

$$f(-x)=-f(x)$$

Hence f(x) in an odd function

43. Evaluate $\lim\limits_{x\to 0} \frac{\sec x-\cos x}{x}$ **(H.W)**

Sol:

$$\lim_{x\to 0} \frac{\sec x-\cos x}{x} \left(\frac{0}{0} \text{ form}\right)$$

$$=\lim_{x\to 0} \frac{\frac{1}{\cos x}-\cos x}{x}$$

$$=\lim_{x\to 0} \frac{\frac{1-\cos^2 x}{\cos x}}{x}$$

$$=\lim_{x\to 0} \frac{\sin^2 x}{x\cos x}$$

$$\lim_{x\to 0} \frac{\sin x}{x}\cdot\frac{\sin^2 x}{\cos x}$$

$$\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\tan x$$

$$\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\tan x$$

$$=1.(0)$$

$$=1.\tan 0=0 \quad \text{Ans.}$$

44. For the function $f(x)=-2x+8$, find $f^{-1}(x)$ (C.W)

Sol: Given $f(x)=-2x+8$, $f^{-1}(x)=?$

Let $f(x)=-2x+8=y$

$\Rightarrow \quad -2x+8=y$

$\Rightarrow \quad -2x=y-8$

$\Rightarrow \quad x=\dfrac{y-8}{-2}$

or

$\Rightarrow \quad x=\dfrac{8-y}{2} \quad \rightarrow$ (i)

Now $y=f(x)$

then $f^{-1}(y)=x$ *from* (i)

$\Rightarrow f^{-1}(y)=\dfrac{8-y}{2}$ Replace "y" by x.

$\Rightarrow f^{-1}(x)=\dfrac{8-x}{2}$ which is required.

45. Evaluate: $\lim\limits_{x\to 3}\dfrac{x-3}{\sqrt{x}-\sqrt{3}}$ (C.W)

Sol: Given $\lim\limits_{x\to 3}\dfrac{x-3}{\sqrt{x}-\sqrt{3}}$ $\left(\frac{0}{0}\text{ form}\right)$

$$=\lim_{x\to 3}\frac{x-3}{\sqrt{x}-\sqrt{3}}\times\frac{\sqrt{x}+\sqrt{3}}{\sqrt{x}+\sqrt{3}}$$

$$=\lim_{x\to 3}\frac{(x-3)(\sqrt{x}+\sqrt{3})}{(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})}$$

$$\because (a-b)(a+b)=a^2-b^2$$

$$=\lim_{x\to 3}\frac{(x-3)(\sqrt{x}+\sqrt{3})}{(\sqrt{x})^2-(\sqrt{3})^2}$$

$$=\lim_{x\to 3}\frac{(x-3)(\sqrt{x}+\sqrt{3})}{(x-3)}$$

$$=\lim_{x\to 3}(\sqrt{x}+\sqrt{3})$$

$$=\sqrt{3}+\sqrt{3}=2\sqrt{3}$$

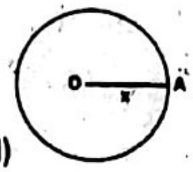
So $\lim_{x\to 3}\frac{x-3}{\sqrt{x}-\sqrt{3}}=2\sqrt{3}$

46. Express the area "A" of a circle as a function of its circumference "C". (H.W)

Sol: Consider a circle with centre at O and radius "x"

Then circumference of circle

i.e $c=2\pi x$



$\Rightarrow \frac{c}{2\pi}=x \rightarrow$ (i)

also

area of circle

i.e

$$A=\pi x^2$$

from (i)

$$A=\pi\left(\frac{c}{2\pi}\right)^2$$

$$A=\frac{\pi c^2}{4\pi^2}$$

$$A=\frac{c^2}{4\pi}$$

Which is required.

47. For the real valued function $f(x)=\frac{2x+1}{x-1}$ find $f^{-1}(x)$ and $f^{-1}(-1)$ (C.W)

Sol:

$f^{-1}(x)=?$ and $f^{-1}(-1)=?$

Given $f(x)=\frac{2x+1}{x-1}$

Let $y=\frac{2x+1}{x-1}$

$\Rightarrow y(x-1)=2x+1$

$\Rightarrow xy-y=2x+1$

$\Rightarrow xy-2x=y+1$

$\Rightarrow x(y-2)=y+1$

$x=\frac{y+1}{y-2}\rightarrow$ (i)

then $y=f(x)$

$\Rightarrow f^{-1}(y)=x$ from (i),

$f^{-1}(y)=\frac{y+1}{y-2}$ Replace "y" by "x"

$$f^{-1}(x)=\frac{x+1}{x-2}$$

Also replace "x" by -1

$$\Rightarrow f^{-1}(-1)=\frac{-1+1}{-1-2}=\frac{0}{-3}=0$$

$$\Rightarrow f^{-1}(-1)=0$$

Which is required

48. $f(x)=\sqrt{x+4}$, find $f\left(x^2+4\right)$

Sol: $f\left(x^2+4\right)=?$

Given $f(x)=\sqrt{x+4}$

Replace x by "x^2+4" we get

$$f\left(x^2+4\right)=\sqrt{x^2+4+4}$$

$f\left(x^2+4\right)=\sqrt{x^2+8}$ which is required.

49. $f(x)=3x^4-2x^2, g(x)=\frac{2}{\sqrt{x}}$ find $f\left(g(x)\right)$

Sol: $f(g(x))=?$

Given $f(x)=3x^4-2x^2$ and $g(x)=\frac{2}{\sqrt{x}}$

$$f(g(x))=f\left(\frac{2}{\sqrt{x}}\right)$$

$$f(g(x))=3\left(\frac{2}{\sqrt{x}}\right)^4-2\left(\frac{2}{\sqrt{x}}\right)^2$$

$$f(g(x))=3\left(\frac{16}{x^2}\right)-\frac{8}{x} \qquad \text{(L.C.M)}$$

$$f(g(x))=\frac{48-8x}{x^2}$$

$$f(g(x))=\frac{8(6-x)}{x^2}$$

Which is required.

50. Evaluate: $\lim\limits_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta}$ **(C.W)**

Sol: Given $\lim\limits_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta}$ $\left(\frac{0}{0}\ form\right)$

$$=\lim_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta}\times\frac{1+\cos\theta}{1+\cos\theta}$$

$$=\lim_{\theta\to 0}\frac{(1-\cos\theta)(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$\because (a-b)(a+b)=a^2-b^2$

$$=\lim_{\theta\to 0}\frac{\left(1^2-\cos^2\theta\right)}{\sin\theta\left(1+\cos\theta\right)}$$

$$=\lim_{\theta\to 0}\frac{\left(1-\cos^2\theta\right)}{\sin\theta\left(1+\cos\theta\right)}$$

$$=\lim_{\theta\to 0}\frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}\left(1+\cos\theta\right)}$$

$$=\lim_{\theta\to 0}\frac{\sin\theta}{1+\cos\theta}$$

$$=\frac{\sin\theta}{1+\cos\theta}=\frac{\theta}{1+1}=\frac{0}{2}=0 \qquad \because \cos\theta=1 \text{ \& } \sin\theta=0$$

So $=\lim_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta}=0$

LONG QUESTION'S OF CHAPTER-1 ACCORDING TO ALP SMART SYLLABUS-2020

1. Solve that parametric equations $x=a\cos\theta, y=b\sin\theta$ represent the equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ **(H.W)**

Topic II: Composition of Functions and Inverse of Function:

2. For the real valued function f, defined below find: **(H.W)**

(i) $f^{-1}(x)$ (ii) $f^{-1}(-1)$ and verify $f(f^{-1}(x))=f^{-1}(f(x))=x$, if $f(x)=(-x+9)^3$

Topic III: Limit of Function:

3. Evaluate $\lim_{\theta\to 0}\frac{1-\cos p\theta}{1-\cos q\theta}$ **(H.W) (2 times)**

4. Evaluate $\lim_{\theta\to 0}\frac{\tan\theta-\sin\theta}{\sin^3\theta}$ **(H.W) (4 times)**

Topic IV: Continux and Discontinux Function:

4. If $f(x)=\begin{cases}3x & \text{if } x\le -2\\ x^2-1 & \text{if } -2<x<2\\ 3 & \text{if } x\ge 2\end{cases}$ discuss continuity at x = 2 and x = -2.

(C.W) (3 times)

5. Discuss the continuity of f(x) at x = 2 if $f(x)=\begin{cases}2x+5 \text{ if } x\le 2\\ 4x+1 \text{ if } x>2\end{cases}$ **(C.W)**

6. If $f(x)=\begin{cases}\dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x\ne 2\\ k & x=2\end{cases}$ find the value of k so that f is continuous at x = 2. **(C.W) (9 times)**

7. Discuss the continuity of $f(x)=\begin{cases}3x-1, & \text{if } x<1\\ 4, & \text{if } x=1\\ 2x, & \text{if } x>1\end{cases}$ at $c=1$ **(H.W)**

Chapter-1 (Examples According to ALP Smart Syllabus)

Example 2: (Page#22) **(C.W)**

Evaluate $\lim\limits_{x\to+\infty}\dfrac{5x^4-10x^2+1}{-3x^3+10x^2+50}$

Sol: Dividing up and down by x^3, we get

$$\lim_{x\to+\infty}\frac{5x^4-10x^2+1}{-3x^3+10x^2+50}=\lim_{x\to+\infty}\frac{5x-10/x+1/x^3}{-3+10/x+50/x^3}$$
$$=\frac{\infty-0+0}{-3+0+0}=\infty$$

Example 4: (Page#22) **(C.W)**

Evaluate (i) $\lim\limits_{x\to-\infty}\dfrac{2-3x}{\sqrt{3+4x^2}}$

(ii) $\lim\limits_{x\to+\infty}\dfrac{2-3x}{\sqrt{3+4x^2}}$

Sol: Here $\sqrt{x^2}=|x|=-x$ as x<0

Dividing up and down by –x, we get

$$\lim_{x\to-\infty}\frac{2-3x}{\sqrt{3+4x^2}}=\lim_{x\to-\infty}\frac{-2/x-3}{\sqrt{3/x^2+4}}=\frac{0+3}{\sqrt{0+4}}=\frac{3}{2}$$

(ii) $\lim\limits_{x\to+\infty}\dfrac{2-3x}{\sqrt{3+4x^2}}=\lim\limits_{x\to+\infty}\dfrac{2/x-3}{\sqrt{3/x^2+4}}=\dfrac{0-3}{\sqrt{0+4}}=\dfrac{-3}{2}$ (Dividing up and down by x)

Example 7: Evaluate: $\lim\limits_{\theta\to0}\dfrac{1-\cos\theta}{\theta}$ **(Page#26)**

Sol: $\lim\limits_{\theta\to0}\dfrac{1-\cos\theta}{\theta}=\dfrac{1-\cos\theta}{\theta}\cdot\dfrac{1+\cos\theta}{1+\cos\theta}$

$$=\frac{1-\cos^2\theta}{\theta(1+\cos\theta)}=\frac{\sin^2\theta}{\theta(1+\cos\theta)}=\sin\theta\left(\frac{\sin\theta}{\theta}\right)\left(\frac{1}{1+\cos\theta}\right)$$

$$\therefore\lim_{\theta\to0}\left(\frac{1-\cos\theta}{\theta}\right)=\lim_{\theta\to0}\sin\theta.\lim_{\theta\to0}\frac{\sin\theta}{\theta}\lim_{\theta\to0}\left(\frac{1}{1+cso\theta}\right)$$

$$=(0)(1)\left(\frac{1}{1+1}\right)=0$$

OBJECTIVES (MCQ'S) OF CHAPTER-2 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Derivative of a function by definition:

1. Notation for derivative was used by Newton is : **(1 time)**

(A) $\frac{dy}{dx}$ (B) Df(x) (C) $f^{0}(x)$ (D) f'(x)

2. $\lim\limits_{\delta x \to 0} \frac{f(x+\delta x)-f(x)}{\delta x} =$ ______ **(6 times)**

(A) $f'(x)$ (B) $f'(a)$ (C) $f'(2)$ (D) $f'(0)$

3. The notation used by Lagrange for derivative is: **(2 times)**

(A) $\frac{df}{dx}$ (B) $f^{\circ}(x)$ (C) $f'(x)$ (D) Df(x)

4. $\lim\limits_{x \to a} \frac{f(x)-f(a)}{x-a} =$ **(2 Times)**

(A) f'(x) (B) f'(a) (C) f' (2) (D) f' (0)

Topic II: Direct Differentiation:

5. $\frac{d}{dx}(\ln 3x) =$ ______ **(3 times)**

(A) $\frac{1}{3x}$ (B) $\frac{3}{x}$ (C) 3x (D) $\frac{1}{x}$

6. The derivative of $\sqrt{x}$ at x = a is : **(2 times)**

(A) $\sqrt{2a}$ (B) $\frac{1}{\sqrt{2a}}$ (C) $\sqrt[2]{a}$ (D) $\frac{1}{2\sqrt{a}}$

7. $\frac{d}{dx}\left(\frac{1}{ax+b}\right)$ **is equal to** **(4 times)**

(A) $\frac{1}{(ax+b)^2}$ (B) $\frac{a}{(ax+b)^2}$ (C) $\frac{-a}{(ax+b)^2}$ (D) $In(ax+b)$

8. If f(x) = $\frac{1}{x^2}$ then f'(x) = ______ **(4 times)**

(A) 1 (B) -1 (C) $\frac{1}{2}$ (D) $\frac{-2}{x^3}$

9. Differential of y is **(5 times)**

(A) dy' (B) $\frac{dy}{dx}$ (C) dx (D) dy

10. If y = $x+\frac{1}{x}$ then $\frac{dy}{dx}$ = ______ **(3 times)**

(A) $1+\frac{1}{x^2}$ (B) $1+\frac{1}{x}$ (C) $1-\frac{1}{x}$ (D) $1-\frac{1}{x^2}$

11. If $y=x^{-3/2}$ then $\frac{dy}{dx}$ is: **(3 times)**

(A) $-\frac{3}{2}x^{-1/2}$ (B) $-\frac{3}{2}x^{1/2}$ (C) $-3x^{5/2}$ (D) $\frac{-3}{2}x^{-5/2}$

12. $\frac{d}{dx}(x^3)$ **is equal to:** **(4 times)**

(A) $\frac{x^3}{3}$ (B) x^2 (C) $3x^2$ (D) $4x^4$

13. $\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right)$ is equal to: (2 times)

(A) $\frac{f(x)g'(x)-g(x)f'(x)}{(f(x))^2}$ (B) $\frac{g(x)f'(x)-f(x)g'(x)}{(f(x))^2}$

(C) $\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$ (D) $\frac{f(x)g'(x)-g(x)f'(x)}{(g(x))^2}$

14. If f(x) = $x^{\frac{2}{3}}$ then $f'(8) =$ ______ (3 times)

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) 3

15. Differential of x^2 is: (6 times)

(A) 2x (B) 2xdx (C) $2x\frac{dy}{dx}$ (D) $2x\frac{dx}{dy}$

16. The derivative of $\frac{x^3+2x^2}{x^3}$ equals: (4 times)

(A) $\frac{2}{x^2}$ (B) $\frac{-2}{x^2}$ (C) $\frac{1}{2x^2}$ (D) $\frac{-1}{2x^2}$

17. If f(x) = x^{100} then $f'(1) =$______ (3 times)

(A) 100 (B) 99 (C) 50 (D) 0

18. $\frac{d}{dx}\left(\frac{x-1}{x}\right)=$______ (4 times)

(A) 1 – 1/x (B) 1 + 1/x (C) $1/x^2$ (D) $1-1/x^2$

19. $\frac{d}{dx}5f(x)$ is equal to. (2 Times)

(a) f' (b) 5 (c) 5 f'(x) (d) f (x)

20. $\frac{d}{dx}(x-\frac{1}{x})$ is equal to.

(a) $1-\frac{1}{x}$ (b) $1+\frac{1}{x}$ (c) $1+\frac{1}{x^2}$ (d) $1-\frac{1}{x^2}$

21. If f(x) = 3 - $\sqrt{x}$ then f' (1) =

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{-1}{2}$ (d) $\frac{-1}{\sqrt{2}}$

22. If y = $x-\frac{1}{x}$, then $\frac{dy}{dx}=$

(a) $1+\frac{1}{x^2}$ (b) $1-\frac{1}{x^2}$ (c) $1+\frac{1}{x}$ (d) $1-\frac{1}{x}$

23. If 3x + 4y – 7 = 0, then $\frac{dy}{dx}$ (2 times)

(a) $\frac{3}{4}$ (b) $\frac{-3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{-4}{3}$

24. $d/_{dx}\left(\frac{1}{\sqrt{x}}\right)=$

(A) $\frac{1}{2x\sqrt{x}}$ (B) $-\frac{1}{2x\sqrt{x}}$ (C) $\frac{1}{2}x\sqrt{x}$ (D) None of these

25. $\frac{d}{dx}\left[\frac{x}{a}\right]=$

(A) $\frac{x}{a^2}$ (B) $\frac{1}{a}$ (C) $\frac{1}{a^2}$ (D) $\frac{x^2}{a^2}$

26. $\frac{d}{dx}(\sqrt{x-9})=$ ________

(A) $\frac{2}{\sqrt{x-9}}$ (B) $\frac{-1}{2\sqrt{x-9}}$ (C) $\frac{1}{2\sqrt{x-9}}$ (D) $\sqrt{x-9}$

27. $\frac{d}{dx}x^n$ is equal to:

(A) nx^{n-1} (B) x^{n-1} (C) $\frac{x^{n+1}}{n}$ (D) nx^{n+1}

28. If f (x+h) = cos (x+h) , then f′ (x) equals:

(A) cos x (B) −cos x (C) sin x (D) - sinx

29. $f(x) = \frac{1}{x-2}$ then $f'(2) =$

(A) 1 (B) 0 (C) -1 (D) ∞

30. $\frac{d}{dx}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 =$: (2 Times)

(A) $1-\frac{1}{2x}$ (B) $1+\frac{1}{x^2}$ (C) 0 (D) $1-\frac{1}{x^2}$

31. $\frac{d}{dx}(x^2+1)^2 =$:

(A) $2(x^2+1)$ (B) $\frac{(x^2+1)^3}{3}$ (C) $2x(x^2+1)$ (D) $4x(x^2+1)$

32. $d/_{dx}\left(3x^{4/3}\right) =$

(A) 4x (B) $4x^{1/3}$ (C) x^4 (D) $4x^{-1/3}$

33. $(f.o\,g)'(x) =$ (2 Times)

(A) $f'(g(x))$ (B) $f(g'(x))$ (C) $f(g(x)).g'(x)$ (D) $f'(g(x)).g'(x)$

34. $\frac{d}{dx}(x^3+4)^{\frac{1}{3}}$ *is equal to:*

(A) $x(x^3+4)^{\frac{-3}{2}}$ (B) $(x^3+4)^{\frac{-2}{3}}2x^2$ (C) $x^2(x^3+4)^{\frac{-2}{3}}$ (D) $(x^3+4)^{\frac{4}{3}}$

35. If $f(x) = \frac{2}{x^3}$ then $f'(2)$ is equal to:

(A) $\frac{3}{8}$ (B) $\frac{5}{8}$ (C) $\frac{1}{4}$ (D) $\frac{-3}{8}$

36. $\frac{d}{dx}\left(\frac{1}{g(x)}\right) =$

(A) $\frac{g'(x)}{g(x)}$ (B) $\frac{-g'(x)}{g(x)}$ (C) $\frac{-g'(x)}{(g(x))^2}$ (D) $\frac{g'(x)}{(g(x))^2}$

Topic III: Chain Rule:

37. $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ is called (5 times)

(A) Product rule (B) Power rule (C) Chain rule (D) Quotient rule

38. Derivative of −sinx w.r.t. sinx is: (2 times)

(A) cosx (B) −cosx (C) 1 (D) -1

39. If y = cos x , u = sin x then $\frac{dy}{du} =$

(A) $\cos x$ (B) $-\cot x$ (C) $-\tan x$ (D) $-\operatorname{cosec} x$

40. Derivative of $(x^3+1)^9$ w.r.t x^3 equals:

(A) $9(x^3+1)^8$ (B) $27x^2(x^3+1)^9$ (C) $3x^2(x^3+1)^8$ (D) $27x(x^3+1)^8$

Topic IV: Derivative of Tringnometric Functions:

41. If $y = \cos\sqrt{x}$, then $\frac{dy}{dx} =$ (2 times)

(A) $-\sin\sqrt{x}$ (B) $\frac{\cos\sqrt{x}}{\sqrt{x}}$ (C) $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$ (D) $\frac{\cos\sqrt{x}}{\sqrt[2]{x}}$

42. $\frac{d}{dx}(\operatorname{cosec}^2 x - \cot^2 x)$ is : (1 time)

(A) $\cot^2 x + \operatorname{cosec}^2 x$ (B) $-2\operatorname{cosec}x\cot x + 2\cot x\operatorname{cosec}^2$

(C) 0 (D) $\sec^2 x + \tan^2$

43. $\frac{d}{dx}\cos 2x$ **is :** **(3 times)**

(A) $-\sin^2 x$ (B) $\sin^2 x$ (C) $-2\sin 2x$ (D) $\sin^2 x$

44. $\frac{d}{dx}(\sqrt{\tan x}) =$ ______ **(6 times)**

(A) $\frac{\sec^2 x}{2\sqrt{\tan x}}$ (B) $\frac{1}{2}\sqrt{\tan x}.\sec^2 x$ (C) $\frac{1}{2}\tan^{\frac{3}{2}} x.\sec^2 x$ (D) $\frac{\sec^2 x}{\sqrt{\tan x}}$

45. $\frac{d}{dx}\sin^{-1}\frac{x}{a} =$ ______ **(3 times)**

(A) $\frac{a}{\sqrt{a^2-x^2}}$ (B) $\frac{-a}{\sqrt{a^2-x^2}}$ (C) $\frac{-1}{\sqrt{a^2-x^2}}$ (D) $\frac{1}{\sqrt{a^2-x^2}}$

46. $\frac{d}{dx}(\tan^{-1} x + \cot^{-1} x)$ **is equal to** **(2 times)**

(A) -1 (B) 0 (C) 1 (D) 2

47. $\frac{d}{dx}(\sin\sqrt{x})$ **is equal to** **(3 times)**

(A) $\cos\sqrt{x}$ (B) $\frac{-\cos\sqrt{x}}{2\sqrt{x}}$ (C) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$ (D) $\frac{\cos\sqrt{x}}{\sqrt{x}}$

48. $\frac{d}{dx}(\cot^{-1} x) =$ ______ **(3 times)**

(A) $\frac{1}{1+x^2}$ (B) $\frac{1}{1-x^2}$ (C) $\frac{-1}{1-x^2}$ (D) $\frac{-1}{1+x^2}$

49. **If f(x) = sin x then** $f'(0) =$ ______ **(2 times)**

(A) 1 (B) -1 (C) 0 (D) x

50. $\frac{d}{dx}(\cot ax) =$ ______ **(1 time)**

(A) $\cos ec^2 ax$ (B) $a\cos ec^2 ax$ (C) $-a\cos ec^2 ax$ (D) $\frac{a\cos ec^2 ax}{a}$

51. $\frac{d}{dx}(\sin^{-1} x)$ **is:** **(11 times)**

(A) $\frac{1}{x\sqrt{x^2-1}}$ (B) $\frac{-1}{x\sqrt{x^2-1}}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{1}{\sqrt{1-x^2}}$

52. $\frac{d}{dx}(\tanh^{-1} x) =$ ______ **(5 times)**

(A) $\frac{1}{1+x^2}$ (B) $\frac{1}{1-x^2}$ (C) $\frac{-1}{1-x^2}$ (D) $\frac{-1}{1+x^2}$

53. $\frac{d}{dx}\left(\frac{1}{\sec x}\right)$ **is equal to:** **(3 times)**

(A) $\frac{d}{dx}\sin x$ (B) $\frac{d}{dx}\cos ecx$ (C) $\frac{d}{dx}\cos x$ (D) $\frac{d}{dx}\cot x$

54. $\frac{d}{dx}(\sin x^3)$ **is equal to:** **(3 times)**

(A) $\cos x^3$ (B) $-\cos x^3$ (C) $x^2 \sin x^3$ (D) $3x^2\cos x^3$

55. $\frac{1}{1+x^2}$ **is the derivative of:** **(4 times)**

(A) $\sin^{-1} x$ (B) $\sec^{-1} x$ (C) $\tan^{-1} x$ (D) $\cot^{-1} x$

56. If $f(x) = \tan^{-1} x$ then $f'(\cot x) =$ ________ **(4 times)**

(A) $\frac{1}{1+x^2}$ (B) $\sin^2 x$ (C) $\cos^2 x$ (D) $\sec^2 x$

57. $\frac{d}{dx}(\sinh 3x) =$ ________ **(6 times)**

(A) 3 sinh 3x (B) 3 cosh 3x (C) cosh 3x (D) sinh 3x

58. If f(x) = sinx then $f'(\pi) =$ ________ **(4 times)**

(A) -1 (B) 0 (C) 1 (D) ½

59. $\frac{d}{dx}(-\cot x)$ equals: **(2 times)**

(A) $\sec^2 x$ (B) $\text{cosec}^2 x$ (C) $-\text{cosec}^2 x$ (D) $-\sec^2 x$

60. $f(x) = \text{Sec}^{-1} x$ then $f'(x) =$ **(2 times)**

(A) $\frac{1}{x\sqrt{x^2-1}}$ (B) Sec x than x (C) $\text{Cos}^2 x$ Cosec x (D) $-\text{Cos}^2 x$ Cosecx

61. $\frac{d}{dx}(\cosh x)$ is equal to. **(3 Times)**

(a) sinh x (b) −sinh x (c) cosech x (d) coth x

62. $\frac{d}{dx}(\cot x)$

(a) cosec x cot x (b) cosec x cot x (c) cosec^2 x (d) $-\text{cosec}^2$x

63. $\frac{d}{dx}(\cosh^{-1} x) =$ **(3 Times)**

(a) $\frac{1}{\sqrt{x^2-1}}$ (b) $\frac{1}{\sqrt{1+x^2}}$ (c) $\frac{-1}{\sqrt{x^2-1}}$ (d) $\frac{1}{\sqrt{1-x^2}}$

64. If f(x) = tan x, then $f'(\frac{\pi}{4})$

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

65. Derivative of cos x^2 w.r.t.x equals. **(2 times)**

(a) 2x $\text{Sin}x^2$ (b) -2x $\text{Sin}x^2$ (c) x $\text{Sin}x^2$ (d) −x $\text{Sin}x^2$

66. If $\sqrt{Cotx} = y$, then $\frac{dy}{dx} =$:

(a) $\frac{\text{cosec}^2 x}{2\sqrt{cotx}}$ (b) $\frac{-\text{cosec}^2 x}{2\sqrt{cotx}}$ (c) $\frac{2\sqrt{cotx}}{\text{cosec}^2 x}$ (d) $\frac{-2\sqrt{cotx}}{\text{cosec}^2 x}$

67. $\frac{d}{dx}(Cosec\,hx)$

(a) cosehx cothx (b) sechx tanhx (c) −cosechx cothx (d) cosech^2x

68. $\frac{d}{dx}[\frac{1}{\sin x}] =$

(a) $\frac{1}{\cos x}$ (b) $-\frac{\sin x}{\cos x}$ (c) $Cosec^2 x$ (d) $-\text{cosec}\,x \cot x$

69. $\frac{1}{2}\frac{d}{dx}(\sin x^2) =$

(a) x $\cos x^2$ (b) $\cos x^2$ (c) 2x $\cos^2$x (d) 2 $\cos x^2$

70. $\frac{d}{dx}(\text{Cos } x^2) =$

(A) -2x $\text{Sin}x^2$ (B) 2x Cos x^2 (C) $4x^2$ Cos x^2 (D) $-x^2$ $\text{Cos}x^2$

71. $\frac{d}{dx}(\text{Sec } x) =$

(A) Secx (B) Cosecx (C) Secxtanx (D) –Secxtanx

72. $\frac{d}{dx}(\text{Cot}^{-1}x)$ =

(A) $\frac{1}{\sqrt{1+x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{x^2-1}}$ (D) $\frac{-1}{1+x^2}$

73. **Derivative of ln (1-cos x) w.r.t x equals**

(A) tanx (B) tanx/2 (C) cotx/2 (D) cotx

74. $\frac{d}{dx}$ **(sin 2x) is equal to**

(A) cos 2x (B) 2cos 2x (C) $\frac{\cos 2x}{2}$ (D) $\frac{-\cos 2x}{2}$

75. $\frac{d}{dx}$ **(sinhx) is equal to :** **(2 Times)**

(A) –coshx (B) coshx (C) tanhx (D) sechx

76. $\frac{d}{dx}$ **(-cosec x) =**

(A) $\cot^2 x$ (B) $\text{cosec}^2 x$ (C) tan x (D) cosec x cot x

77. **If y = sec** $\left(\frac{3\pi}{2}-x\right)$ **, then** y_1 **equals:**

(A) cosec x cot x (B) –cosec x cot x (C) sec x tan x (D) –sec x tanx

78. **f(x) = cotx then f′(π/6) =**

(A) -4 (B) 4 (C) 1/4 (D) -1/4

79. $\frac{d}{dx}\cot^2 2x =$ **(2 Times)**

(A) 4 cot 2x cosec 2x (B) -4 cot 2x cosec^2 2x (C) 4 $\cot^2$ 2x cosec 2x (D) -4 cot 2x

80. *If* $f(x) = \cos x$ *then* $f'\left(\frac{\pi}{2}\right)$ =:

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) -1

81. $\frac{d}{dx}(\tan h^{-1} x)$ = **(2 times)**

(A) $\frac{1}{1+x^2}$ (B) $\frac{1}{1-x^2}$ (C) $\frac{1}{x^2-1}$ (D) $\frac{-1}{1+x^2}$

82. **If y = sin**$^{-1}\sqrt{x}$ **, then** $\frac{dy}{dx}$ **equals.**

(A) $\frac{1}{2\sqrt{x}\sqrt{1-x^2}}$ (B) $\frac{-1}{2\sqrt{x}\sqrt{1-x^2}}$ (C) $\frac{1}{2\sqrt{x}\sqrt{1-x}}$ (D) $\frac{1}{\sqrt{x}\sqrt{1-x}}$

83. $\frac{d}{dx}(\cos^{-1}x)$ **is equal to:**

(A) $\frac{1}{\sqrt{1-x^2}}$ (B) $\frac{-1}{\sqrt{1-x^2}}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-1}{1+x^2}$

84. $\frac{d}{dx}$ (sec hx) **is equal to:**

(A) $\sec x \tan x$ (B) $-\sec x \tan x$ (C) $-secc\, hx \tan hx$ (D) $\sec hx \tan hx$

85. $\frac{d}{dx}(\sec x) =$ **(2 Times)**

(A) $\sec x \tan x$ (B) $-\sec x \tan x$ (C) $\sec^2 x$ (D) $\sec x \tan^2 x$

86. $\frac{d}{dx}(\cos h^{-1}x) =$

(A) $\frac{1}{\sqrt{1+x^2}}$ (B) $\frac{-1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{x^2-1}}$ (D) $\frac{-1}{\sqrt{x^2-1}}$

Topic V: Derivative of exponential and Logrithmic Function:

87. **If** $y = \ell n(f(x))$**, then** $\frac{dy}{dx}$ = **(6 times)**

(A) $\frac{f'(x)}{f(x)\ln a}$ (B) $\frac{f'(x)}{f(x)}$ (C) $f(x)$ (D) $\frac{1}{f(x)}$

88. $\frac{d}{dx}[\ell n(\ell n x)] =$ **(5 times)**

(A) $\frac{1}{x}$ (B) $\frac{1}{x\ell na}$ (C) $\frac{1}{x\ell nx}$ (D) $\frac{x}{\ell nx}$

89. $\frac{d}{dx}a^x =$______ **(3 times)**

(A) a^x (B) $a^x \ln a$ (C) $\frac{a^x}{\ln a}$ (D) $\frac{1}{a^x \ln a}$

90. $\frac{d}{dx}\log_a x =$______ **(5 times)**

(A) $\frac{1}{x}$ (B) $\frac{1}{x}\log a$ (C) $\frac{1}{x\ln a}$ (D) $\frac{1}{x\ln e}$

91. If y = $\ell n(\tan hx)$ then $\frac{dy}{dx}$ is : **(5 times)**

(A) $\sec h^2 x \coth x$ (B) $2\operatorname{cosech}2x$ (C) $\sec hx \coth^2 x$ (D) $-2\sec hx \coth x$

92. $\frac{d}{dx}\left(2^{\sqrt{x}}\right)$ **equals:** **(3 times)**

(A) $2^{\sqrt{1}}$ (B) $2^{\sqrt{x}}\ln 2$ (C) $\frac{2^{\sqrt{x}}\ln 2}{2\sqrt{x}}$ (D) $\frac{2^{\sqrt{1}}}{2\sqrt{x}}$

93. $\frac{d}{dx}(\ell nx)$ **is equal to:** **(5 times)**

(A) $\frac{1}{\ell nx}$ (B) $\frac{1}{x}$ (C) x (D) ℓnx

94. If y = e^{3x} then y_3 is: **(1 time)**

(A) e^{3x} (B) e^3 (C) $9e^{3x}$ (D) $27e^{3x}$

95. The derivative of f(x) = e^x equals: **(2 times)**

(A) e^x (B) xe^{x-1} (C) $\frac{e^{x-1}}{x-1}$ (D) $\frac{e^{x+1}}{x+1}$

96. If f(x) = ln (x + 1) then $f'(x)$ =______ **(2 times)**

(A) x + 1 (B) $\frac{1}{1-x}$ (C) $\frac{1}{x+1}$ (D) 1 − x

97. $\frac{d}{dx}(e^{\ell nx^2}) =$

(a) $e^{\ell nx^2}$ (b) e^{2x} (c) $\frac{2e^{\ell nx^2}}{x}$ (d) $2x^2$

98. The differential co-efficient of $e^{\sin x}$ is: **(3 Times)**

(a) $e^{\sin x}.\cos x$ (b) $e^{\sin x}.\sin x$ (c) $e^{\cos x}.\cos x$ (d) $\sin x.e^{\sin x-1}$

99. If y = $\ell n(\sin x)$, then $\frac{dy}{dx}$ equals:

(a) tan x (b) cot x (c) – tan x (d) – cot x

100. $\frac{d}{dx}e^{f(x)} equals$ **(2 times)**

(a) $e^{f'(x)}$ (b) $e^{f'(x)}.f'(x)$ (c) $\frac{f'(x)}{e^e}$ (d) $\frac{e^{f(x)}}{e'(x)}$

101. $\frac{d}{dx}(\ln e^x) =$ **(2 times)**

(a) 2x (b) 1 (c) x^2 (d) $\frac{1}{x^2}$

102. $\frac{d}{dx}(e^{\tan x}) =$

(a) $e^{\tan x}$ (b) $e^{\tan x \ln \sec^2 x}$ (c) $e^{\tan x}\sec^2 x$ (d) $e^{\tan x}\ln\tan x$

103. $\frac{d}{dx}\left(e^{x^2+1}\right) =$

(A) e^{x^2+1} (B) $2xe^{x^2+1}$ (C) $2e^{x^2+1}$ (D) $-2xe^{x^2+1}$

104. $y = 5e^{3x-4}$ then $\frac{dy}{dx} =$ **(2 times)**

(A) $\frac{5}{3}e^{3x-4}$ (B) $15e^{3x-4}$ (C) $-5e^{3x-4}$ (D) $15e^{3x-4}$

105. If $f(x) = e^{ax}$ then $f'(x)$ is equal to:

(A) $\frac{e^{ax}}{a}$ (B) $-\frac{e^{ax}}{a}$ (C) ae^{ax} (D) $-ae^{ax}$

106. $\frac{d}{dx}(e^{\cos x})$ equals: **(2 Times)**

(A) $-\sin x e^{\cos x}$ (B) $\sin x e^{\cos x}$ (C) $\cos x e^{\sin x}$ (D) $-\cos x e^{\sin x}$

107. $\frac{d}{dx}\left(((\ell n x)^m)^k\right)$:

(A) $\frac{mk}{x}(\ell n\, x)^{mk-1}$ (B) $\frac{k}{x^m}\ (\ell n\, x)^{k-1}$ (C) $\frac{1}{x^{mk}}$ (D) $\frac{mk}{x}$

108. $\frac{d}{dx}(\log_a^x)$

(A) $\frac{1}{x}$ (B) $\frac{1}{x}.\frac{1}{lna}$ (C) $-\frac{1}{x\, lna}$ (D) $-\frac{1}{x}$

109. $\frac{d}{dx}\left(e^{\sqrt{x}}\right) =$

(A) $e^{\sqrt{x}}$ (B) $\frac{-e^{\sqrt{x}}}{\sqrt{x}}$ (C) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$ (D) $\frac{\sqrt{x}\, e^{\sqrt{x}}}{2}$

110. $\frac{d}{dx} In\left(\frac{1}{x}\right) =$

(A) x (B) -x (C) $\frac{1}{x}$ (D) $-\frac{1}{x}$

111. $\frac{d}{dx}(a^x) =$

(A) $a^x \ln a$ (B) $\frac{1}{In\, a}a^x$ (C) $a^x \ln e$ (D) $-a^x \ln a$

112. $\frac{d}{dx}a^{\lambda x} =$ **(2 times)**

(A) $\lambda a^{\lambda x}\ell na$ (B) $a^{\lambda x}\ell na$ (C) $\frac{a^{\lambda x}}{\ell na}$ (D) $\frac{a^{\lambda x}}{\lambda}$

113. $\frac{d}{dx}\left((\ell n\,(e^x + e^{-x})\right) =$

(A) $\frac{e^x+e^{-x}}{e^x-e^{-x}}$ (B) $\frac{e^x-e^{-x}}{e^x+e^{-x}}$ (C) $\frac{e^x-e^{-x}}{-e^x+e^{-x}}$ (D) $\frac{-e^x+e^{-x}}{e^x-e^{-x}}$

114. $\frac{d}{dx}a^{f(x)}$ equals:

(A) $a^{f(x)}.f'(x)$ (B) $a^{f(x)}.f'(x).a$ (C) $a^{f(x)}$ (D) $a^{f(x)}f'(x)\ell na$

Topic VI: Higher Derivative:

115. If y = sinx then $\frac{d^2y}{dx^2}$ equals

(a) cos x (b) – cosx (c) y (d) – y

116. If $y = e^{2x}$ then y_4 equals:-

(A) $16e^{2x}$ (B) $8e^{2x}$ (C) $2e^{2x}$ (D) e^{2x}

117. If $y = e^{ax}$ then $y_4 =$

(A) a^4e^{ax} (B) $2\frac{e^{ax}}{a}$ (C) $3\,e^{ax}$ (D) xe^{ax}

118. Let y = cos (ax + b) , then y_2 equals:

(A) ay (B) -ay (C) a^2y (D) $-a^2y$

Topic VII: Series Expansions of Function:

119. $1+x+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\ldots$ is Maclaurin's expansion of: **(1 time)**

(A) cosx (B) six (C) e^x (D) lnx

120. Maclaurins expansion of ln (1 + x) is : **(3 times)**

A) $x-\frac{x^3}{3!}+\frac{x^5}{5!}+...$ (B) $1-\frac{x^2}{2!}+\frac{x^4}{4!}+...$ (C) $-x-\frac{x^2}{2!}-\frac{x^3}{3!}+...$ (D) $x-\frac{x^2}{2}+\frac{x^3}{3}+...$

121. The series $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+...=$:

(A) cosx (B) tanx (C) sinx (D) -sinx

122. $a_0+a_1x+a_2x^2+...a_nx^n+...$ is:

(A) Maclaurin's series (B) Taylor Series (C) Power Series (D) Binomial Series

Topic VIII: Increasing and Decreasing Functions:

123. For a stationary point for a function f, we have $f'(x)$ = **(3 times)**

(A) 0 (B) +ve (C) –ve (D) ∞

124. The function $f(x)=2+3x^2$ has minimum value at : **(4 times)**

(A) X = 3 (B) x = 2 (C) x = 1 (D) x = 0

125. f(x) increases if: **(1 time)**

(A) $f'(x)<0$ (B) $f'(x)>0$ (C) $f'(x)=0$ (D) $f'(x)\geq 0$

126. If $f'(c)=0$ then f has relative maximum at x = c if : **(3 times)**

(A) $f''(c)>0$ (B) $f''(c)<0$ (C) $f''(c)=0$ (D) $f''(c)\geq 0$

127. The function $f(x)=3x^2$ has minimum value at : **(2 times)**

(A) x = 3 (B) x = 2 (C) x = 1 (D) x = 0

128. If $f'(c)=0$ and $f''(c)<0$ then f (x) will give at x = c

(a) Maximum value (b) Minimum value

(c) Neither maximum nor minimum value (d) Stationary Value

129. The minimum value of the function $f(x)=x^2+2x-3$ is at x =.

(a) -3 (b) 1 (c) 0 (d) -1

130. $f(x)=-3x^2$ has maximum value at:

(a) x = -2 (b) x = - 1 (c) x = 0 (d) x = 1

131. Solution of differential equation, $\frac{dy}{dx}=y$ is: **(2 Times)**

(A) ce^x (B) ce^{-x} (C) e^x (D) e^{-x}

132. f (x) = sin x is decreasing function in the interval:

(A) $(-\pi, -\pi/2)$ (B) $(-\pi/2, 0)$ (C) $(0, \pi/2)$ (D) $(-3\pi/2, -2\pi)$

133. Let "f" be differential function in neighbourhood of "c" where $f'(c)=0$, then "f" has relative maxima at x = c if:

(A) $f''(c)=0$ (B) $f''(c)>0$ (C) $f''(c)<0$ (D) $f''(c)$ does not exist

134. The critical value of $f(x)=x^2-x-2$ equals:

(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$ (C) 2 (D) -2

135- $\frac{d}{dx}\cos^2 x$ is equal to:

(A) $-\sin^2 x$ (B) 2sinx (C) 2sinxcosx (D) – 2cosxsinx

136- $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+...$ is Maclaurin series of:

(A) e^x (B) sinx (C) cosx (D) $\ell n(1+x)$

137- If $x=at^2$, y = 2at, then $\frac{dy}{dx}$ is equal to:

(A) t (B) $\frac{1}{t}$ (C) t^2 (D) $\frac{1}{t^2}$

138- $\frac{d}{dx}\left(\frac{1}{ax+b}\right)$ is equal to:

(A) ax + b (B) $\frac{-1}{(ax+b)^2}$ (C) $\frac{-a}{(ax+b)^2}$ (D) $\ell n(ax+b)$

139- If y = sin3x, then y_2 is equal to:

(A) 9sin3x (B) -9sin3x (C) 9cos3x (D) -9cos3x

140- If $y=e^{f(x)}$ then $y' =$:

(A) $e^{f'(x)}.f(x)$ (B) $e^{f(x)}.f'(x)$ (C) $e^{f'(x)}.\log f(x)$ (D) $e^{f'(x)}.f'(x)$

141- For relative maxima at x = c:

(A) f(c) < f(x) (B) f(c) > f(x) (C) f(c) ≥ f(x) (D) f(c) ≤ f(x)

142- If $f'(a-\varepsilon)<0$ and $f'(a+\varepsilon)<0$ then at x =a f(x) has:

(A) Relative Minima (B) Relative Maxima (C) Point of Inflexion (D) Critical Point

143- $\frac{1}{2}\frac{d}{dx}\left[Tan^{-1}x-Cot^{-1}x\right]=$:

(A) $\frac{-1}{1+x^2}$ (B) $\frac{1}{1+x^2}$ (C) $\frac{1}{1-x^2}$ (D) $\frac{-1}{1-x^2}$

144- $\frac{d}{dx}[g(x)]^{-1} =$:

(A) $-[g(x)]^{-2}$ (B) $-[g'(x)]^{-2}$ (C) $-g'(x)[g(x)]^{-2}$ (D) $\frac{-g(x)}{[g(x)]^2}$

145- $\frac{d}{dx}(cosec\,x)=$

(A) $-cosec^2x$ (B) $-cosecx\ Cotx$ (C) $-cosec^2x\ Cotx$ (D) $-cot^2x$

146- $\frac{d}{dx}\left(a^{\sqrt{x}}\right)=$:

(A) $a^{\sqrt{x}}.\ln a$ (B) $\frac{-a^{\sqrt{x}}}{\ln a}$ (C) $\frac{a^{\sqrt{x}}.\ln a}{2\sqrt{x}}$ (D) $\frac{a^{\sqrt{x}}}{2\sqrt{x}\ln a}$

147- Geometrically $\frac{dy}{dx}$ means:

(A) Tangent of slope (B) Slope of tangent (C) Slope of line (D) Slope of x-axis

148- If $y=\sqrt{1-x^2}$, $0<x<1$ *then* $\frac{dy}{dx}=$:

(A) $\sqrt{x^2-1}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{x}{\sqrt{1-x^2}}$ (D) $\frac{-x}{\sqrt{1-x^2}}$

149- If $y=e^{\sin x}$, *then* $\frac{dy}{dx}=$:

(A) e^{sinx} (B) $e^{sinx}cosx$ (C) $e^{sinx}+cosx$ (D) $-e^{sinx}cosx$

150- If f(x) has second derivative at "c" such that $f'(c)=0$ and $f''(c)<0$ then "c" is a point of:

(A) Maxima (B) Minima (C) Zero point (D) Point of inflection

151- $\frac{d}{dx}\sin^{-1}x =$

(A) $\frac{1}{\sqrt{1+x^2}}$ (B) $\cos^{-1}x$ (C) $\frac{1}{\sqrt{1-x^2}}$ (D) $\frac{-1}{\sqrt{1-x}}$

152- $\frac{d}{dx}(5^x)=$ **(2 times)**

(A) 5^x (B) $5^x\ell n5$ (C) $\frac{5^x}{\ell n5}$ (D) $5(5^x)$

153- $\frac{d}{dx}\left(\sec^{-1}x+cosec^{-1}x\right)=$

(A) 1 (B) −1 (C) 0 (D) 2

154- $\frac{d}{dx}\left(\frac{1}{x^2}\right)$ at $x=1$ is =: **(2 times)**

(A) -2 (B) 2 (C) 1 (D) -1

155. $\frac{d}{dx}e^{f(x)} =$ **(2 times)**

(a) $e^{f(x)}$ (b) $f(x)e^{f(x)}$ (c) $e^{f(x)}f'(x)$ (d) $f(x)e^{f(x)-1}$

156. $\frac{d}{dx}\sqrt{x} =$ **(2 times)**

(a) $\sqrt{1}$ (b) 1 (c) $\frac{1}{2}\sqrt{x}$ (d) $\frac{1}{2\sqrt{x}}$

157. If f(x) has maximum value at x = c, then f'(c) = 0 but f'' (c) is:

(a) Negative (b) Positive (c) Zero (d) Undefined

158. $\frac{d}{dx}\left(\frac{a}{x}\right) =$

(a) a (b) $\frac{1}{x}$ (c) $\frac{a}{x^2}$ (d) $-\frac{a}{x^2}$

159. $\frac{d}{dx}(Cosx^2) =$

(a) $\sin x^2$ (b) $-\sin x^2$ (c) $2x\sin x^2$ (d) $-2x\sin x^2$

160. **If** $y = \sin^{-1}\frac{x}{a}$, **then** $\sin y =$:

(a) $\cos y$ (b) $\cos x$ (c) $\frac{x}{a}$ (d) $\frac{y}{a}$

161. $\frac{d}{dx}(f(u)) =$:

(a) $f'(u)$ (b) $f(du)$ (c) $f'(u)\frac{du}{dx}$ (d) $f'(u)du$

162. $\frac{d}{dx}(x-5)(3-x) =$

(a) $2x+8$ (b) $-2x+8$ (c) $2x-8$ (d) $x+8$

163. $\frac{d}{dx}(2x^2+3)^5 =$

(a) $(2x^2+3)^4 20x$ (b) $20(2x^2+3)^5$ (c) $15(2x^2+3)^5$ (d) $(2x^2+3)^5 100x$

164. The Derivative of x^3 w.r.t x^2 is equal to:

(a) $\frac{3x^2}{2}$ (b) $\frac{3x}{2}$ (c) $\frac{2}{3x}$ (d) $\frac{2}{3x^2}$

165. Second term in Maclaurin Series expansion of $f(x) = e^x$ is

(a) 1 (b) x^2 (c) x (d) x^3

166. $\frac{d}{dx}(ln\,2x) =$

(a) $\frac{1}{2x}$ (b) $\frac{1}{x}$ (c) $-\frac{1}{2x}$ (d) $2x$

167. $y = \sin 3x$ **then** y_2 **is**

(a) $9\cos x$ (b) $-9\sin 3x$ (c) $9\sin 3x$ (d) $-9\cos 3x$

ANSWERS TO THE MULTIPLE CHIOCE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14
c	a	c	b	d	d	c	d	d	d	d	c	a	c
15	16	17	18	19	20	21	22	23	24	25	26	27	28

b	b	a	c	c	c	c	a	b	b	b	c	a	d
29	30	31	32	33	34	35	36	37	38	39	40	41	42
d	d	d	b	d	c	d	c	c	d	c	a	c	c
43	44	45	46	47	48	49	50	51	52	53	54	55	56
c	a	d	b	c	d	a	c	d	b	C	d	c	b
57	58	59	60	61	62	63	64	65	66	67	68	69	70
b	a	b	a	a	d	a	b	b	b	c	d	a	a
71	72	73	74	75	76	77	78	79	80	81	82	83	84
c	d	c	b	b	d	a	a	b	d	b	c	b	c
85	86	87	88	89	90	91	92	93	94	95	96	97	98
a	c	b	c	b	c	b	c	b	d	a	c	c	a
99	100	101	102	103	104	105	106	107	108	109	110	111	112
b	b	b	c	b	b	c	a	a	b	c	d	a	a
113	114	115	116	117	118	119	120	121	122	123	124	125	126
b	d	d	a	a	d	c	d	c	ç	a	d	b	b
127	128	129	130	131	132	133	134	135	136	137	138	139	140
d	a	d	c	a	a	C	a	d	a	b	c	b	b
141	142	143	144	145	146	147	148	149	150	151	152	153	154
C	b	b	c	b	c	b	d	b	a	c	b	c	a
155	156	157	158	159	160	161	162	163	164	165	166	167	
c	d	a	d	d	c	c	b	a	b	c	b	b	

SHORT QUESTION'S OF CHAPTER-2 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Derivative of a function by definition:

1. Define derivative of the function **(5 times)**

Sol: Derivative of the function:

Let f(x) be any function then if $\lim\limits_{x\to 0}\dfrac{f(x+h)-f(x)}{h}$ exist is called derivative of the f(x). It is denoted by f′(x).

2. Differentiate $\dfrac{2x-3}{2x+1}$ w.r.t. 'x' **(H.W) (6 times)**

Sol: Let $y=\dfrac{2x-3}{2x+1}$

Diff.w.r.t. 'x'

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{2x-3}{2x+1}\right)$$

$$=\frac{(2x+1)2-(2x-3)2}{(2x+1)^2}$$

$$=\frac{4x+2-4x-6}{(2x+1)^2}$$

$$=\frac{8}{(2x+1)^2}$$

3. **Differentiate** $\sqrt{x+\sqrt{x}}$ **w.r.t. 'x'** **(C.W) (9 times)**

Sol: Let y = $\sqrt{x+\sqrt{x}}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{x+\sqrt{x}}\right)$$

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x+\sqrt{x}}}\cdot\frac{d}{dx}\left(x+\sqrt{x}\right)=\frac{1}{2\sqrt{x+\sqrt{x}}}\cdot\left(1+\frac{1}{2\sqrt{x}}\right)$$

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x+\sqrt{x}}}\left(\frac{2\sqrt{x}+1}{2\sqrt{x}}\right)=\frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$$

4. **Find** $\frac{dx}{dt}$ **and** $\frac{dy}{dt}$ **when** $x=at^2$ **, y = 2at.** **(C.W)**

Sol:

$x=at^2$

Diff. w.r.t. 't'

$$\frac{dx}{dt}=2at$$

y = 2at

Diff. w.r.t. 't'

$$\frac{dy}{dt}=2a$$

5. **Find** y_1 **, if** $x^3-y^3=a^3$ **(C.W) (3 times)**

Sol: Diff. w.r.t. 'x'

$$3x^2-3y^2\frac{dy}{dx}=0$$

Dividing by 3

$$x^2-y^2\frac{dy}{dx}=0$$

$$y^2\frac{dy}{dx}=x^2 \quad , \quad \frac{dy}{dx}=\frac{x^2}{y^2}$$

6. **Find** $\frac{dy}{dx}$ **if y = x cos y** **(H.W) (6 times)**

Sol: y = x cos y

$$\frac{dy}{dx}=x(-\sin y)\frac{dy}{dx}+\cos y(1)$$

$$\frac{dy}{dx}+x\sin y\frac{dy}{dx}=\cos y$$

$$\frac{dy}{dx}(1++x\sin y)=\cos y$$

$$\frac{dy}{dx}=\frac{\cos y}{1+x\sin y}$$

7. **Find** $\frac{dy}{dx}$ **if** $y^2+x^2-4x=5$ **(C.W) (2 times)**

Sol. $y^2+x^2-4x=5$

Diff. w.r.t. 'x'

$$\frac{d}{dx}(y^2+x^2-4x)=\frac{d}{dx}(5)$$

$$2y\frac{dy}{dx}+2x-4=0$$

$$2y\frac{dy}{dx}=4-2x=\frac{4-2x}{2y}=\frac{2(2-x)}{2y}=\frac{2-x}{y}$$

8: **Find $\frac{dy}{dx}$ if $3x+4y+7=0$** **(H.W)** **(4 times)**

Sol: Given

$3x+4y+7=0$

Diff. w.r.t x

$\frac{dy}{dx}(3x+4y+7)=0$

$3\frac{d}{dx}(x)+4\frac{d}{dx}(y)+\frac{d}{dx}7=0$

$3(1)+4\frac{dy}{dx}+0=0$

$3+4\frac{dy}{dx}=0$

$4\frac{dy}{dx}=-3$

$\frac{dy}{dx}=\frac{-3}{4}$

9: **Find $\frac{dy}{dx}$ if $y=\frac{x}{lnx}$** **(H.W)** **(7 times)**

Sol: Let $y=\frac{x}{\ell nx}$

Diff w.r.t x

$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x}{\ell nx}\right)$ $\qquad \because \left(\frac{u}{v}\right)'=\frac{vu'-uv'}{v^2}$

$\frac{dy}{dx}=\frac{(\ell nx)\frac{d}{dx}(x)-x\frac{d}{dx}\ell nx}{(\ell nx)^2}$

$=\frac{\ell nx\,(1)-x\frac{1}{x}}{(lnx)^2}$ $\qquad \because \frac{d}{dx}\ell nx=\frac{1}{x}$

$\frac{dy}{dx}=\frac{\ell nx-1}{(\ell nx)^2}$

10: **Find $\frac{dy}{dx}$ when $y=\sqrt{x}-\frac{1}{\sqrt{x}}$** **(H.W)** **(3 times)**

Sol: Let $y=\sqrt{x}-\frac{1}{\sqrt{x}}$

$y=x^{1/2}-x^{-1/2}$

Diff. w.r.t.x

$\frac{dy}{dx}=\frac{d}{dx}x^{1/2}-\frac{d}{dx}x^{-1/2}$

$\frac{dy}{dx}=\frac{1}{2}x^{-1/2}-\left(-{}^{1}\!/_{2}\right)x^{-3/2}$

$=\frac{1}{2\sqrt{x}}+\frac{1}{2x^{3/2}}$

$=\frac{1}{2\sqrt{x}}+\frac{1}{2x\sqrt{x}}=\frac{x+1}{2x\sqrt{x}}$

11: **If $y=x^4+2x^2+2$ then prove that $\frac{dy}{dx}=4x\sqrt{y-1}$** **(H.W)** **(5 times)**

Sol: $y=x^4+2x^2+2$

Diff. w.r.t.x

$\frac{dy}{dx} = \frac{d}{dx}(x^4 + 2x^2 + 2)$

$\frac{dy}{dx} = 4x^3 + 4x + 0$

$\frac{dy}{dx} = 4x^3 + 4x$

$\frac{dy}{dx} = 4x(x^2 + 1)$

$\frac{dy}{dx} = 4x\sqrt{(x^2+1)^2} = 4x\sqrt{x^4 + 2x^2 + 1}$

$\frac{dy}{dx} = 4x\sqrt{x^4 + 2x^2 + 2 - 1} = 4x\sqrt{y-1}$ (Hence Proved)

12: **If $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$, find $\frac{dy}{dx}$** **(C.W) (8 times)**

Sol: $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

$y = (\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2\sqrt{x}.\frac{1}{\sqrt{x}}$

$y = x + \frac{1}{x} - 2$

$y = x + x^{-1} - 2$

Diff w.r.t. x

$\frac{dy}{dx} = \frac{d}{dx}(x + x^{-1} - 2)$

$= \frac{d}{dx}x + \frac{d}{dx}x^{-1} - \frac{d}{dx}2$

$\frac{dy}{dx} = 1 + (-1)x^{-2} - 0 = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$

13: **Differentiate w.r.t. x $\frac{x^2+1}{x^2-3}$.** **(H.W) (5 times)**

Sol: Let $y = \frac{x^2+1}{x^2-3}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x^2-3}\right)$

$\frac{dy}{dx} = \frac{(x^2-3)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^2-3)}{(x^2-3)^2}$

$\frac{dy}{dx} = \frac{(x^2-3)(2x+0) - (x^2+1)(2x-0)}{(x^2-3)^2}$

$\frac{dy}{dx} = \frac{2x\left[x^2-3-x^2-1\right]}{(x^2-3)^2}$

$\frac{dy}{dx} = \frac{2x(-4)}{(x^2-3)^2} = \frac{-8x}{(x^2-3)^2}$

14. **Find derivative of $\sqrt{\frac{a-x}{a+x}}$ w.r.t. x** **(H.W) (2 times)**

Sol **Given**

$$y = \sqrt{\frac{a-x}{a+x}}$$

Differentiate w.r.t x.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a-x}{a+x}\right)^{1/2} \qquad \because \frac{d}{dx}f(x)^n = nf(x)^{n-1}.\frac{d}{dx}f(x)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{a-x}{a+x}\right)^{1/2-1} \times \frac{d}{dx}\left(\frac{a-x}{a+x}\right)$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{a-x}{a+x}\right)^{-1/2}\times\frac{(a+x)\frac{d}{dx}(a-x)-(a-x)\frac{d}{dx}(a+x)}{(a+x)^2}\qquad \because\left(\frac{u}{v}\right)'=\frac{vu'-uv'}{v^2}$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{a+x}{a-x}\right)^{1/2}\times\frac{(a+x)(0-1)-(a-x)(0+1)}{(a+x)^2}$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{a+x}{a-x}\right)^{1/2}\times\frac{(a+x)(-1)-(a-x)(1)}{(a+x)^2}$$

$$\frac{dy}{dx}=\frac{1}{2(a-x)^{1/2}(a+x)^{-1/2}}\times\frac{-a-x-a+x}{(a+x)^2}$$

$$\frac{dy}{dx}=\frac{-2a}{2(a-x)^{1/2}(a+x)^{2-1/2}}$$

$$\frac{dy}{dx}=\frac{-a}{\sqrt{a-x}\;(a+x)^{3/2}}$$

15. Find the derivative of $y = x^3 + 2x + 3$. **(C.W)**

Sol **Given**

$$y=x^3+2x+3$$

Differentiate w.r.t. x

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^3+2x+3\right)\qquad \because \frac{d}{dx}x^n=nx^{n-1}$$

$$=\frac{d}{dx}x^3+2\frac{d}{dx}.(x)+\frac{d}{dx}.(3)\qquad \because \frac{d}{dx}c=0$$

$$=3x^2+2(1)+0$$

$$\frac{dy}{dx}=3x^2+2$$

Topic III: Chain Rule:

16. Differentiate sin x w.r.t. cot x. **(H.W) (4 times)**

Sol: Let

$y = \sin x$ $\quad$ $u = \cot x$

Diff. w.r.t. 'x' $\quad$ Diff. w.r.t. 'x'

$$\frac{dy}{dx}=\cos x\qquad \frac{du}{dx}=-\operatorname{cosec}^2 x$$

By chain rule

$$\frac{dy}{du}=\frac{dy}{dx}.\frac{dx}{du}$$

$$\frac{dy}{du}=\cos x\left(\frac{1}{-\operatorname{cosec}^2 x}\right)=-\sin^2 x\cos x$$

17. Find $\frac{dy}{dx}$ if $x = 1-t^2$, $y = 3t^2-2t^3$ **(C.W) (4 times)**

Sol: $x = 1-t^2$ $\quad$ $y = 3t^2-2t^3$

Diff. w.r.t. 't' $\quad$ Diff. w.r.t. 't'

$$\frac{dx}{dt}=\frac{d}{dt}(1-t^2)\qquad \frac{dy}{dt}=\frac{d}{dt}(3t^2-2t^3)$$

$\frac{dx}{dt}=-2t$ $\frac{dy}{dt}=3(2t)-2(3t)^2=6t-6t^2$

By chain rule

$\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}$

$\frac{dy}{dx}=\frac{6t(1-t)}{-2t}$

$\frac{dy}{dx}=-3(1-t)$

18: Differentiate $\sin^2 x$ w.r.t $\cos^4 x$ **(C.W) (6 times)**

Sol: Let

$y = \sin^2 x.$ w.r.t. $z = \text{Cos}^4 x.$

Diff. w.r.t. x

$\frac{dy}{dx}=\frac{d}{dx}\sin^2 x$

$\frac{dy}{dx}=2\sin x\cos x$

Diff. w.r.t. x

$\frac{dz}{dx}=\frac{d}{dx}\text{Cos}^4 x$

$= 4\cos 3\ x.\ (-\sin x)$

$\frac{dz}{dx}=-4\cos^3 x\sin x$

$\frac{dz}{dx}=\frac{-1}{4\text{Cos}^3 x\text{Sin}x}$

Using Chain Rule

$\frac{dy}{dz}=\frac{dy}{dx}\times\frac{dx}{dz}$

$\frac{dy}{dx}=\frac{2\sin x\cos x}{-4\cos^2 x\sin x}=\frac{1}{-2\cos^2 x}=\frac{-1}{2}\sec^2 x$

Topic IV: Derivative of Tringnometric Functions:

19. Find $\frac{d}{dx}\cot^{-1}\frac{x}{a}$ **(H.W) (4 times)**

Sol: $\frac{d}{dx}\cot^{-1}\frac{x}{a}$

$=\frac{-1}{1+\left(\frac{x}{a}\right)^2}.\frac{d}{dx}\left(\frac{x}{a}\right)$

$=\frac{-1}{1+\frac{x^2}{a^2}}\left(\frac{1}{a}\right)$

$=\frac{-1}{\frac{a^2+x^2}{a^2}}\left(\frac{1}{a}\right)$

$=\frac{-a^2}{a^2+x^2}\left(\frac{1}{a}\right)$

$=\frac{-\cancel{a^2}}{a^2+x^2}\left(\frac{1}{\cancel{a}}\right)=\frac{-a}{a^2+x^2}$

20. Find $\frac{dy}{dx}$ if $y=\tanh(x)^2$ (C.W) **(2 times)**

Sol: $y=\tanh x^2$

Diff. w.r.t. 'x'

$$\frac{dy}{dx}=\frac{d}{dx}(\tanh x^2)$$

$$\frac{dy}{dx}=\text{sech}^2 x^2 .2x$$

$$\frac{dy}{dx}=2x\,\text{sech}^2 x^2$$

21. **If tan y(1 + tan x) = 1 – tan x, than show that $\frac{dy}{dx}=-1$** **(H.W)**

Sol: tan y(1 + tan x) = 1 – tan x

$$\tan y=\frac{1-\tan x}{1+\tan x}$$

$$\tan y=\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x} \qquad \because \tan\frac{\pi}{4}=1$$

$$\tan y=\tan\left(\frac{\pi}{4}-x\right)$$

$\Rightarrow y=\frac{\pi}{4}-x$ Diff. w.r.t. 'x'

$$\Rightarrow\frac{dy}{dx}=\frac{d}{dx}\left(\frac{\pi}{4}\right)-\frac{d}{dx}x$$

$$\frac{dy}{dx}=-1$$

22: **Differentiate w.r.t x, $\cos\sqrt{x}+\sqrt{\sin x}$** **(C.W) (2 times)**

Sol: $y=\cos\sqrt{x}+\sqrt{sinx}$

Diff. w.r.t.x

$$\frac{dy}{dx}=\frac{d}{dx}\cos\sqrt{x}+\frac{d}{dx}\sqrt{sinx}$$

$$\frac{dy}{dx}=-\sin\sqrt{x}\times\frac{1}{2}x^{-1/2}+\frac{1}{2}(\sin x)^{-1/2}\cos x.$$

$$\frac{dy}{dx}=\frac{-\sin\sqrt{x}}{2\sqrt{x}}+\frac{\cos x}{2\sqrt{\sin x}}$$

23: **Differentiate y = sin h^{-1} (x^3) w.r.t.x** **(C.W) (4 times)**

Sol: Let y = Sin h^{-1} (x^3)

Diff. w.r.t.x

$$\frac{dy}{dx}=\frac{d}{dx}\text{Sinh}^{-1}(x^3) \qquad \because \frac{d}{dx}\text{Sinh}^{-1}x=\frac{1}{\sqrt{1+x^2}}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1+(x^3)^2}}\times\frac{d}{dx}x^3=\frac{3x^2}{\sqrt{1+x^6}}$$

24: **Prove that $\frac{d}{dx}(\sinh^{-1}x)=\frac{1}{\sqrt{1+x^2}}$** **(C.W) (3 times)**

Sol: Let y = Sinh^{-1} x
Then Sin h y = x

Diff. w.r.t.x $\qquad \because \frac{d}{dx}\sinh x=\cosh x$

$\cosh y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{1+\sinh^2 y}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$

Hence

$\frac{d}{dx}\sinh^{-1}x = \frac{1}{\sqrt{1+x^2}}$

25. Differentiate $\tan^3\theta \sec^2\theta$ with respect to θ. (H.W) (2 times)

Sol Given

$$y = \tan^3\theta\sec^2\theta$$

Differentiate w.r.t θ

$$\frac{dy}{d\theta} = \frac{d}{dx}\left(\tan^3\theta\sec^2\theta\right) \qquad \because (u.v)' = uv' + vu'$$

$$\because \frac{d}{d\theta}\tan\theta = \sec^2\theta$$

$$\frac{dy}{d\theta} = \tan^3\theta\frac{d}{dx}\sec^2\theta + \sec^2\theta\frac{d}{dx}\tan^3\theta \qquad \because \frac{d}{d\theta}\sec\theta = \sec\theta\tan\theta$$

$$\frac{dy}{d\theta} = \tan^3\theta.2\sec\theta\sec\theta\tan\theta + \sec^2\theta 3\tan^2\theta\sec^2\theta$$

$$\frac{dy}{d\theta} = 2\tan^4\theta sec^2\theta + 3\tan^2\theta\sec^4\theta$$

$$\frac{dy}{dx} = \tan^2\theta\sec^2\theta\left(2\tan^2\theta + 3\sec^2\theta\right)$$

Topic V: Derivative of exponential and Logrithmic Function:

26. If $y = e^{-2x}\sin 2x$, then find $\frac{d^2y}{dx^2}$ (H.W) (4 times)

Sol: $y = e^{-2x}\sin 2x$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(e^{-2x}\sin 2x)$$

$$\frac{dy}{dx} = e^{-2x}.\frac{d}{dx}(\sin 2x) + (\sin 2x)\frac{d}{dx}e^{-2x}$$

$$\frac{dy}{dx} = e^{-2x}(2\cos 2x) + (\sin 2x)(-2e^{-2x})$$

$$\frac{dy}{dx} = 2e^{-2x}\cos 2x - 2e^{-2x}\sin 2x$$

$$\frac{dy}{dx} = 2e^{-2x}(\cos 2x - \sin 2x)$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}2e^{-2x}(\cos 2x - \sin 2x)$$

$$\frac{dy}{dx} = 2e^{-2x}\frac{d}{dx}(\cos 2x - \sin 2x) + (\cos 2x - \sin 2x)\frac{d}{dx}\left(2e^{-2x}\right)$$

$$\frac{d^2y}{dx^2}=2e^{-2x}(-2\sin 2x-2\cos 2x)+(-4)e^{-2x}(\cos 2x-\sin 2x)$$

$$\frac{d^2y}{dx^2}=-4e^{-2x}(\cancel{\sin 2x}+\cos 2x+\cos 2x-\cancel{\sin 2x})$$

$$\frac{d^2y}{dx^2}=-8e^{-2x}\cos 2x$$

27. If $y=ln\tanh x$, then find $\frac{dy}{dx}$ **(H.W) (3 times)**

Sol: $y=ln(\tanh x)$

Diff. w.r.t. 'w'

$$\frac{dy}{dx}=\frac{d}{dx}ln(\tanh x)$$

$$\frac{dy}{dx}=\frac{1}{\tanh x}.\sec h^2x=\frac{1}{\frac{\sinh x}{\cosh x}}\times\frac{1}{\cosh^2 x}$$

$$\frac{dy}{dx}=\frac{1}{\sinh x.\cosh x}=\frac{2}{2\sinh x.\cosh x}=\frac{2}{\sinh 2x}$$

$$\frac{dy}{dx}=2\operatorname{cosech}2x$$

28. If $y=x^2e^{-x}$, then find y_1 **(C.W)**

Sol: $y=x^2e^{-x}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx}=\frac{d}{dx}(x^2e^{-x})=x^2\frac{d}{dx}(e^{-x})+(e^{-x})\frac{d}{dx}(x^2)$$

$$\frac{dy}{dx}=x^2(-e^{-x})+(e^{-x})(2x)$$

$$\frac{dy}{dx}=2xe^{-x}-x^2e^{-x}=xe^{-x}(2-x)$$

29. If $y=xe^{\sin x}$, then find $\frac{dy}{dx}$ **(H.W) (6 times)**

Sol: $y=xe^{\sin x}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx}=x\frac{d}{dx}e^{\sin x}+e^{\sin x}\frac{d}{dx}(x)$$

$$\frac{dy}{dx}=xe^{\sin x}\cos x+e^{\sin}=e^{\sin x}(1+x\cos x)$$

30. If $y=x^2\ln\frac{1}{x}$ then find $\frac{dy}{dx}$ **(H.W) (4 times)**

Sol: $y=x^2ln\frac{1}{x}$

$$y=x^2ln\,x^{-1}=-x^2ln\,x$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx}=\frac{-d}{dx}(x^2 lnx)$$

$$\frac{dy}{dx}=-\left[x^2\frac{d}{dx}(\ln x)+(\ln x)\frac{d}{dx}(x^2)\right]$$

$$\frac{dy}{dx}=-\left[x^2\left(\frac{1}{x}\right)+2xlnx\right]$$

$$\frac{dy}{dx}=-(x+2xlnx)$$

$$\frac{dy}{dx}=-x(1+2\ell nx)$$

31. **If** $y=5e^{3x-4}$ **then find** $\frac{dy}{dx}$ **(H.W) (2 times)**

Sol: $y=5e^{3x-4}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx}=\frac{d}{dx}5e^{3x-4}$$

$$\frac{dy}{dx}=5\frac{d}{dx}(e^{3x-4})$$

$$\frac{dy}{dx}=5e^{3x-4}.\frac{d}{dx}(3x-4)$$

$$\frac{dy}{dx}=5e^{3x-4}(3)$$

$$\frac{dy}{dx}=15e^{3x-4}$$

32. **Find** $\frac{dy}{dx}$ **if y =** $\log_{10}(ax^2+bx+c)$ **(C.W)**

Sol: y = $\log_{10}(ax^2+bx+c)$

$$y=\frac{\ln(ax^2+bx+c)}{\ln 10}$$

$$\frac{dy}{dx}=\frac{1}{\ln 10}.\frac{d}{dx}\ln(ax^2+bx+c)$$

$$\frac{dy}{dx}=\frac{1}{\ln 10}.\frac{1}{(ax^2+bx+c)}.\frac{d}{dx}(ax^2+bx+c)$$

$$\frac{dy}{dx}=\frac{1}{\ln 10(ax^2+bx+c)}.(2ax+b)$$

$$\frac{dy}{dx}=\frac{2ax+b}{\ln 10(ax^2+bx+c)}$$

33. **Find** $f'(x)$ **if f(x) =** $\ln\sqrt{e^{2x}+e^{-2x}}$ **(HC.W)(2 times)**

Sol: y = $\ln\sqrt{e^{2x}+e^{-2x}}$ = $\frac{1}{2}\ln(e^{2x}+e^{-2x})$

Diff. w.r.t. 'x' on both sides

$$\frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}\ell n(e^{2x}+e^{-2x})$$

$$=\frac{1}{2}\frac{1}{\left(e^{2x}+e^{-2x}\right)}\times\frac{d}{dx}(e^{2x}+e^{-2x})$$

$$=\frac{1}{\left(2e^{2x}+e^{-2x}\right)}\times 2e^{2x}-2e^{-2x}$$

$$=\frac{\cancel{2}(e^{2x}-e^{-2x})}{\cancel{2}(e^{2x}+e^{-2x})}$$

$$=\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$$

34: **Find f′ (x) if f(x) = $e^{\sqrt{x}-1}$** **(H.W) (4 times)**

Sol: Let $f(x)=e^{\sqrt{x}-1}$

$f'(x)=\frac{d}{dx}e^{\sqrt{x}-1}$

$f'(x)=e^{\sqrt{x}-1}\times\frac{d}{dx}(\sqrt{x}-1)$

$f'(x)=e^{\sqrt{x}-1}\left[\frac{1}{2\sqrt{x}}-0\right]=\frac{e^{\sqrt{x}-1}}{2\sqrt{x}}$

35. **Find $\frac{dy}{dx}$ if $y=\ln\sqrt{\frac{x^2-1}{x^2+1}}$** **(3 times)**

Sol **Given**

$$y=\ln\sqrt{\frac{x^2-1}{x^2+1}}$$

$$y=\ln\left(\frac{x^2-1}{x^2+1}\right)^{1/2}$$

$$y=\frac{1}{2}\ln\left(\frac{x^2-1}{x^2+1}\right)$$

$$y=\frac{1}{2}\left[\ln\left(x^2-1\right)-\ln\left(x^2+1\right)\right]$$

Differentiate w.r.t x

$$\frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}\left[\ln\left(x^2-1\right)-\ln\left(x^2+1\right)\right]$$

$$\frac{dy}{dx}=\frac{1}{2}\left[\frac{2x}{x^2-1}-\frac{2x}{x^2+1}\right]$$

$$\frac{dy}{dx}=\frac{2x}{2}\left[\frac{1}{x^2-1}-\frac{1}{x^2+1}\right]$$

$$\frac{dy}{dx}=x\left[\frac{x^2+1-\left(x^2-1\right)}{\left(x^2-1\right)\left(x^2+1\right)}\right]$$

$$\frac{dy}{dx}=\frac{x\left(x^2+1-x^2+1\right)}{x^4-1}$$

$$\frac{dy}{dx}=\frac{2x}{x^4-1}$$

36. **Find $f'(x)$ if $f(x)=\sqrt{\ln\left(e^{2x}+e^{-2x}\right)}$** **(3 times)**

Sol **Given**

$$f(x)=\sqrt{\ln\left(e^{2x}+e^{-2x}\right)}$$

Differentiate w.r.t. x

$$f'(x)=\frac{d}{dx}\left[\ln\left(e^{2x}+e^{-2x}\right)\right]^{1/2}$$

$$f'(x)=\frac{1}{2}\left[\ln\left(e^{2x}+e^{-2x}\right)\right]^{1/2-1}\times\frac{d}{dx}\ln\left(e^{2x}+e^{-2x}\right)$$

$$f'(x)=\frac{1}{2}\left[\ln\left(e^{2x}+e^{-2x}\right)\right]^{-1/2}\times\frac{1}{\left(e^{2x}+e^{-2x}\right)}\times\frac{d}{dx}\left(e^{2x}+e^{-2x}\right)$$

$$f'(x)=\frac{1}{2\ln\sqrt{\left(e^{2x}+e^{-2x}\right)}}\times\frac{1}{\left(e^{2x}+e^{-2x}\right)}\times\left(e^{2x}\times 2+e^{-2x}(-2)\right)$$

$$f'(x)=\frac{2\left(e^{2x}-e^{-2x}\right)}{2\sqrt{\ln\left(e^{2x}+e^{-2x}\right)}\left(e^{2x}+e^{-2x}\right)}$$

$$f'(x)=\frac{e^{2x}-e^{-2x}}{\sqrt{\ln\left(e^{2x}+e^{-2x}\right)}\left(e^{2x}+e^{-2x}\right)}$$

37. **Find $\frac{dy}{dx}$ if $y=\ln(9-x^2)$.** **(H.W) (3 times)**

Sol **Given**

$$y=\ln\left(9-x^2\right)$$

Differentiate w.r.t. x

$$\frac{dy}{dx}=\frac{d}{dx}\ln\left(9-x^2\right)$$

$$\because \frac{d}{dx}\ln f(x)=\frac{1}{f(x)}\times f'(x)$$

$$\frac{dy}{dx}=\frac{1}{9-x^2}\times\frac{d}{dx}\left(9-x^2\right)$$

$$\frac{dy}{dx}=\frac{1}{9-x^2}\times(0-2x)$$

$$\frac{dy}{dx}=\frac{-2x}{9-x^2}\text{ Ans.}$$

Topic VI: Higher Derivative:

38. **If $2x^5-3x^4+4x^3+x-2$, then find y_2** **(C.W) (2 times)**

Sol: $y = 2x^5-3x^4+4x^3+x-2$

Diff. w.r.t. 'x'

$$\frac{dy}{dx}=10x^4-12x^3+12x^2+1$$

$$y_1=10x^4-12x^3+12x^2+1$$

Diff. w.r.t. 'x'

$$\frac{dy_1}{dx}=40x^3-36x^2+24x$$

$$y_1 = 40x^3 - 36x^2 + 24x$$

39. **Find y_2 if $y = x^2 e^x$.** **(C.W) (2 times)**

Sol **Given**

$$y = x^2 . e^{-x}$$

Differentiate w. r. t x

$$y_1 = \frac{d}{dx}x^2e^{-x} \qquad \because (uv)' = uv' + vu'$$

$$= x^2\frac{d}{dx}e^{-x} + e^{-x}\frac{d}{dx}x^2 \qquad \because \frac{d}{dx}x^n = nx^{n-1}$$

$$= x^2e^{-x}\left(\frac{d}{dx}(-x)\right) + e^{-x}2x$$

$$= x^2e^{-x}(-1) + e^{-x}2x$$

$$y_1 = e^{-x}(-x^2 + 2x)$$

Again differentiate w. r. t. x

$$y_2 = \frac{d}{dx}\ e^{-x}(-x^2 + 2x)$$

$$y_2 = e^{-x}\frac{d}{dx}(-x^2 + 2x) + (-x^2 + 2x)\frac{d}{dx}e^{-x}$$

$$= e^{-x}(-2x + 2) + (-x^2 + 2x)\ e^{-x}(-1)$$

$$= e^{-x}[-2x + 2 + x^2 - 2x]$$

$$y_2 = e^{-x}[x^2 - 4x + 2] \qquad \text{Ans.}$$

Topic VII: Series Expansions of Function:

40. **Write Maclaurin's series expansion.** **(C.W) (6 times)**

Sol: The Maclaurin's series expansion of a function f(x) is given by

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + ...$$

41: **Expand cos x by Maclaurin's series expansion.** **(C.W) (3 times)**

Sol: Let f(x) = xos x. $\Rightarrow$ f(0) = cos 0 = 1

F′ (x) = - sin x $\Rightarrow$ f′ (0) = -sin 0 = 0

F″ (x) =- cos x $\Rightarrow$ f″ (0) = -cos0 = -1

F‴ (x) = sinx $\Rightarrow$ f‴ (0) = sin0 = 0

Using Maclaurin's Series.

$$f(x) = f(0) + x\,f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + ...$$

$$\cos x = 1 + x(0) + \frac{x^2}{2!}(-1) + \frac{x^3}{3!}(0) + ...$$

$$\cos x = 1 + 0 - \frac{x^2}{2!} + 0 + \frac{x^4}{4!} + ...$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ---$$

Topic VIII: Increasing and Decreasing Functions:

42: **Find the interval for which function is increasing and decreasing $f(x) = 4 - x^2$ for $e \in (-2, 2)$.** **(H.W) (4 times)**

Sol: Given $f(x) = 4 - x^2$

Diff. w.r.t.x

$f'(x) = -2x$

If f is increasing then, If f is decreasing then

f' (x) > 0	f' (x) < 0
-2 x > 0	-2 x < 0
x < 0	x > 0
x ∈ [-2 , 0[	x ∈] 0 , 2]
So f increase on [-2 , 0 [	, So f decreases on] 0 , 2]

43. Determine the intervals in which $f(x)=\cos x \;: x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ is increasing or decreasing function. **(C.W) (3 times)**

Sol **Given**

$$f(x)=\cos x \qquad : x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$$

Differentiate w.r.t x

$$f'(x)=\frac{d}{dx}\cos x$$

$$f'(x)=-\sin x$$

x ∈ I & IV quadrants

We know that

sinx < 0 in quad IV

So $f'(x)=-\sin x>0$ in quad IV

$$f'(x)=-\sin x>0 \text{ for } x\in\left(\frac{-\pi}{2},0\right)$$

Hence $f(x)=\cos x$ is increasing on $\left(\frac{-\pi}{2};0\right)$

Also we know that $\sin x>0$ in I quad.

So $f'(x)=-\sin x<0$ for $x\in\left(0,\frac{\pi}{2}\right)$

Hence $f'(x)=\cos x$ is decreasing on the interval $\left(0,\frac{\pi}{2}\right)$

44. Determine the intervals in which f is increasing or decreasing $f(x)=\sin x \quad x\in[-\pi,\pi]$ **(H.W) (2times)**

Sol **Given**

$$f(x)=Sinx \qquad \because x\in[-\pi,\pi]$$

$$f'(x)=Cosx$$

Now

$$f'(x)=0$$

$$Cosx=0$$

$$x=Cos^{-1}(0) \quad\Rightarrow\quad x=\frac{-\pi}{2},\frac{\pi}{2}$$

So intervals : $\left[-\pi,\frac{-\pi}{2}\right],\left[\frac{-\pi}{2},\frac{\pi}{2}\right],\left[\frac{\pi}{2},\pi\right]$

$f(x)=Sinx$ is increasing on $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

i-e $\quad f'(x)=Cosx > 0 \quad , \quad \forall\; x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

Now $f'(x) = Cosx < 0$ $\forall\ x \in \left(-\pi, \frac{-\pi}{2}\right)$

$f(x) = Sinx$ is decreasing on $\left(-\pi, \frac{-\pi}{2}\right)$

And $f'(x) = Cosx < 0$; $\forall\ x \in \left(\frac{\pi}{2}, \pi\right)$

So $f(x) = Sinx$ is decreasing on $\left(\frac{\pi}{2}, \pi\right)$ Ans.

45. Find $\frac{dy}{dx}$ if $y = (3x^2 - 2x + 7)^6$ **(H.W)**

Sol: $y = (3x^2 - 2x + 7)^2$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}(3x^2 - 2x + 7)^6$$

$$\because \frac{d}{dx} f(x)^n = nf(x)^{n-1} \times \frac{d}{dx} f(x)$$

$$\frac{dy}{dx} = 6\,(3x^2 - 2x + 7)^{6-1} \times \frac{d}{dx}(3x^2 - 2x + 7)$$

$$\frac{dy}{dx} = 6\,(3x^2 - 2x + 7)^5 (6x - 2)$$

$$\frac{dy}{dx} = 12\,(3x^2 - 2x + 7)^5 (3x - 1)$$ Ans.

46. Differentiate w.r.t. x, $y = \frac{2x-1}{\sqrt{x^2+1}}$ **(H.W)**

Sol: $y = \frac{2x-1}{\sqrt{x^2+1}}$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x-1}{\sqrt{x^2+1}}\right)$$

$$\because \left(\frac{u}{v}\right)^1 = \frac{vu^1 - uv^1}{v^2}$$

$$\frac{dy}{dx} = \frac{\sqrt{x^2+1}\,\frac{d}{dx}(2x-1) - (2x-1)\,\frac{d}{dx}\sqrt{x^2+1}}{\left(\sqrt{x^2+1}\right)^2}$$

$$\frac{dy}{dx} = \frac{\sqrt{x^2+1}\,(2-0) - (2x-1)\,\frac{1}{2}\,(x^2+1)^{1/2-1}(2x+0)}{(x^2+1)}$$

$$\frac{dy}{dx} = \frac{\sqrt{x^2+1}\,(2-0) - (2x-1)\,\frac{1}{2}\,(x^2+1)^{1/2-1}(2x+0)}{(x^2+1)}$$

$$\frac{dy}{dx} = \frac{2\sqrt{x^2+1} - 2x\,(2x-1)\,\frac{1}{2}\,(x^2+1)^{-1/2}}{(x^2+1)}$$

$$\frac{dy}{dx}=\frac{2\sqrt{x^2+1}-2x\,(2x-1)\,\frac{1}{2}\,(x^2+1)^{-1/2}}{(x^2+1)}$$

$$\frac{dy}{dx}=\frac{2\sqrt{x^2+1}-(2x^2-x)\frac{1}{\sqrt{x^2+1}}}{(x^2+1)}$$

$$\frac{dy}{dx}=\frac{\frac{2.\left(\sqrt{x^2+1}\right)^2-2x^2+x}{\sqrt{x^2+1}}}{(x^2+1)}$$

$$\frac{dy}{dx}=\frac{2(x^2+1)-2x^2+x}{(x^2+1)\sqrt{x^2+1}}$$

$$\frac{dy}{dx}=\frac{2x^2+2-2x^2+x}{(x^2+1)^{1+1/2}}$$

$$\frac{dy}{dx}=\frac{2+x}{(x^2+1)^{3/2}}$$ Ans.

47. Differentiate $Cos^{-1}\left(\frac{x}{a}\right)$ w.r.t. x **(C.W) (2 times)**

Sol: Let $y=Cos^{-1}\left(\frac{x}{a}\right)$

Diff. w.r.t. x

$$\frac{dy}{dx}=\frac{d}{dx}Cos^{-1}\left(\frac{x}{a}\right)$$

$$\because \frac{d}{dx}Cos^{-1}x=\frac{-1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\times\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-\frac{x^2}{a^2}}}\times\frac{1}{a}$$

$$\frac{dy}{dx}=\frac{-1}{\sqrt{\frac{a^2-x^2}{a^2}}}\times\frac{1}{a}$$

$$\frac{dy}{dx}=\frac{-1}{\frac{\sqrt{a^2-x^2}}{a}}\times\frac{1}{a}$$

$$\frac{dy}{dx}=\frac{-1}{\sqrt{a^2-x^2}}$$

48. Find by definition, the derivative of $2-\sqrt{x}$ w.r.t x. **(H.W)**

Sol: Let $y=2-\sqrt{x}$ ____________ (1)

$$y+\delta y = 2-\sqrt{x+\delta x} \quad \text{\_\_\_\_\_\_\_\_\_\_(2)}$$

Eq (2) – Eq (1)

$$y+\delta y - y = 2-\sqrt{x+\delta x}-2+\sqrt{x}$$

$$\delta y = -\sqrt{x+\delta x}+\sqrt{x}$$

$$\delta y = \sqrt{x}-\sqrt{x+\delta x}$$

$$\delta y = \left(\sqrt{x}-\sqrt{x+\delta x}\right)\frac{\left(\sqrt{x}+\sqrt{x+\delta x}\right)}{\sqrt{x}+\sqrt{x+\delta x}}$$ $\because$ by rationalization

$$\delta y = \frac{\left(\sqrt{x}\right)^2-\left(\sqrt{x+\delta x}\right)^2}{\sqrt{x}+\sqrt{x+\delta x}}$$

$$= \frac{x-x-\delta x}{\sqrt{x}+\sqrt{x+\delta x}}$$

$$\delta y = \frac{-\delta x}{\sqrt{x}+\sqrt{x+\delta x}}$$

Dividing by δx on both sides

$$\frac{\delta y}{\delta x} = \frac{-1}{\sqrt{x}+\sqrt{x+\delta x}}$$

Taking $\lim\limits_{\delta x\to 0}$ on both sides

$$\lim_{\delta x\to 0}\frac{\delta y}{dx} = \lim_{x\to 0}\frac{-1}{\sqrt{x}+\sqrt{x+\delta x}}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{x}+\sqrt{x+0}}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{x}+\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$$

49. Differentiate $(\ln x)^x$ w.r.t. x. **(C.W) (2 times)**

Sol: Let $y=(\ln x)^x$ $\quad\because \ln a^b = b\ln a$

$$\ln y = x\ln(\ln x)$$

Diff. w.r.t. x

$$\frac{d}{dx}\ln y = \frac{d}{dx}\, x\,\ln(\ln x)$$

$\because (u.v)' = uv' + vu'$

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\ln(\ln x)+\ln(\ln x)\frac{d}{dx}.\,x$$

$$\because \frac{d}{dx}\ln f(x) = \frac{1}{f(x)}.f'(x)$$

$$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{\ln x}\frac{d}{dx}\ln x + \ln(\ln x).1$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{\ln x}.\frac{1}{x}+\ln(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\ln x}+\ln(\ln x)$$

$$\frac{dy}{dx}=y\left[\frac{1}{\ln x}+\ln(\ln x)\right]$$

$$\frac{dy}{dx}=(\ln x)^x\left[\ln(\ln x)+\frac{1}{\ln x}\right]$$

50. **Find** $f'(x)$ **if** $f(x)=\ln\left(e^x+e^{-x}\right)$ **(H.W)**

Sol: Let $f(x)=\ln\left(e^x+e^{-x}\right)$

Diff. w.r.t. x

$$f'(x)=\frac{d}{dx}\ln\left(e^x+e^{-x}\right)$$

$$=\frac{1}{\left(e^x+e^{-x}\right)}\times\frac{d}{dx}\left(e^x+e^{-x}\right)$$

$$\because \frac{d}{dx}\ln(f(x))=\frac{1}{f(x)}\times f'(x)$$

$$=\frac{1}{\left(e^x+e^{-x}\right)}\times\left(e^x+e^{-x}(-1)\right) \qquad \because \frac{d}{dx}e^{f(x)}=e^{f(x)}.f'(x)$$

$$f'(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

51. **Find y_2 if** $y=\sqrt{x}+\frac{1}{\sqrt{x}}$ **(H.W) (3 times)**

Sol:

$$y=\sqrt{x}+\frac{1}{\sqrt{x}}$$

$$y=(x)^{1/2}+(x)^{-1/2}$$

Diff. w.r.t. x

$$y_1=\frac{d}{dx}\left(x^{1/2}+x^{1/2}\right)$$

$$y_1=\frac{d}{dx}x^{1/2}+\frac{d}{dx}x^{-1/2} \qquad \because \frac{d}{dx}x^n=n\,x^{n-1}$$

$$y_1=\frac{1}{2}x^{1/2-1}+\left(\frac{-1}{2}\right)x^{-1/2-1}$$

$$y_1=\frac{1}{2}x^{-1/2}-\frac{1}{2}x^{-3/2}$$

$$y_1=\frac{1}{2}\left(x^{-1/2}-x^{-3/2}\right)$$

Again diff. w.r.t. x

$$y_2=\frac{1}{2}\frac{d}{dx}\left(x^{-1/2}-x^{-3/2}\right)$$

$$y_2=\frac{1}{2}\left(\frac{-1}{2}x^{-1/2-1}-\left(\frac{-3}{2}\right)x^{-3/2-1}\right)$$

$$y_2=\frac{1}{2}\left(\frac{-1}{2}x^{-3/2}+\frac{3}{2}x^{-5/2}\right)$$

$$y_2=\frac{1}{4}\left(\frac{-1}{x^{3/2}}+\frac{3}{x^{5/2}}\right)$$

$$y_2 = \frac{1}{4}\left(\frac{-x+3}{x^{5/2}}\right)$$

$$y_2 = \frac{3-x}{4x^{5/2}}$$

52. **Find** $\frac{dy}{dx}$ **if** $x^2 - 4xy - 5y = 0$ **(H.W) (2 times)**

Sol: $x^2 - 4xy - 5y = 0$

Diff. w.r.t. x

$$\frac{d}{dx}\left(x^2 - 4xy - 5y\right) = \frac{d}{dx}(0)$$

$$\frac{d}{dx}x^2 - 4\frac{d}{dx}(xy) - 5\frac{dy}{dx} = 0 \qquad \because \frac{d}{dx}x^n = nx^{n-1}$$

$$2x - 4\left[x\frac{dy}{dx} + y\frac{d}{dx}x\right] - 5\frac{dy}{dx} = 0$$

$$2x - 4\left[x\frac{dy}{dx} + y\right] - 5\frac{dy}{dx} = 0$$

$$2x - 4x\frac{dy}{dx} - 4y - 5\frac{dy}{dx} = 0$$

$$(2x - 4y) - (4x + 5)\frac{dy}{dx} = 0$$

$$(4x + 5)\frac{dy}{dx} = 2x - 4y$$

$$\frac{dy}{dx} = \frac{2x - 4y}{4x + 5}$$

$$\frac{dy}{dx} = \frac{2(x - 2y)}{4x + 5} \qquad \text{Ans.}$$

53. **Differentiate** $x^2 - \frac{1}{x^2}$ **w.r.t.** x^4 **(H.W)**

Sol: Let $y = x^2 - \frac{1}{x^2}$, $t = x^4$ Find $\frac{dy}{dt}$

$y = x^2 - x^{-2}$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 - x^{-2}\right)$$

$$\because \frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{dy}{dx} = 2x - (-2)x^{-2-1}$$

$$\frac{dy}{dx} = 2x + 2x^{-3}$$

$$\frac{dy}{dx} = 2\left(x + x^{-3}\right)$$

$$\frac{dy}{dx} = 2\left(x + \frac{1}{x^3}\right)$$

$$\frac{dy}{dx}=2\left(\frac{x^4+1}{x^3}\right)$$

Now $\quad t=x^4$

Diff. w.r.t. x

$$\frac{dt}{dx}=\frac{d}{dx}x^4$$

$$\frac{dt}{dx}=4x^3$$

$$\frac{dx}{dt}=\frac{1}{4x^3}$$

Applying chain Rule

$$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$$

$$\frac{dy}{dt}=2\left(\frac{x^4+1}{x^3}\right)\times\frac{1}{4x^3}$$

$$\frac{dy}{dt}=\frac{x^4+1}{x^6}$$

54. Differentiate $Sin^{-1}\sqrt{1-x^2}$ w.r.t. x. **(H.W) (2 times)**

Sol:

Let $\quad y=Sin^{-1}\sqrt{1-x^2}$

Diff. w.r.t x

$$\frac{dy}{dx}=\frac{d}{dx}Sin^{-1}\sqrt{1-x^2}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-\left(1-x^2\right)^2}}\times\frac{d}{dx}\sqrt{1-x^2} \qquad \because \frac{d}{dx}Sin^{-1}x=\frac{1}{\sqrt{1-x^2}}\times\frac{dx}{dx}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-1+x^2}}\times\frac{1}{2}\left(1-x^2\right)^{\frac{1}{2}-1}\times(0-2x)$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{x^2}}\times\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}(-2x)$$

$$\frac{dy}{dx}=\frac{-x}{x\sqrt{1-x^2}}$$

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-x^2}}$$

55. Find $\frac{dy}{dx}$ if $y=\ln\left(x+\sqrt{x^2+1}\right)$ **(H.W) (2 times)**

Sol: $\quad y=\ln\left(x+\sqrt{x^2+1}\right)$

Diff. w.r.t. x

$$\frac{dy}{dx}=\frac{d}{dx}\ln\left(x+\sqrt{x^2+1}\right)$$

$$\frac{dy}{dx}=\frac{1}{\left(x+\sqrt{x^2+1}\right)}\times\frac{d}{dx}\left(x+\sqrt{x^2+1}\right) \qquad \because \frac{d}{dx}\ln f(x)=\frac{1}{f(x)}\cdot f'(x)$$

$$\because \frac{d}{dx}x^n=nx^{n-1}$$

$$\frac{dy}{dx}=\frac{1}{\left(x+\sqrt{x^2+1}\right)}\times\left(1+\frac{1}{2}\left(x^2+1\right)^{\frac{1}{2}-1}(2x+0)\right)$$

$$\frac{dy}{dx}=\frac{1}{\left(x+\sqrt{x^2+1}\right)}\times\left(1+\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}(2x)\right)$$

$$\frac{dy}{dx}=\frac{1}{\left(x+\sqrt{x^2+1}\right)}\times\left(1+\frac{x}{\sqrt{x^2+1}}\right)$$

$$\frac{dy}{dx}=\frac{1}{\left(x+\sqrt{x^2+1}\right)}\times\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{x^2+1}}$$

56. Differentiate $(\ln x)^x$ w.r.t. x **(C.W)**

Sol: $\frac{d}{dx}(\ln x)^x=?$

Let $y=(\ln x)^x \quad \rightarrow$ (i)

Taking natural logarithm on both sides.

$\ln y=\ln(\ln x)^x$

$\ln y=x\ln(\ln x)$ Diff w.r.t. x

$$\frac{d}{dx}\ln y=\frac{d}{dx}\left(x\ln(\ln x)\right)$$

$$\frac{1}{y}\frac{dy}{dx}=x\frac{d}{dx}\ln(\ln x)+\ln(\ln x)\frac{d}{dx}x$$

$$\frac{1}{y}\frac{dy}{dx}=x\left[\frac{1}{\ln x}\frac{d}{dx}(\ln x)\right]+\ln(\ln x).1$$

$$\frac{1}{y}\frac{dy}{dx}=\cancel{x}\left[\frac{1}{\ln x}.\frac{1}{\cancel{x}}\right]+\ln(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{\ln x}+\ln(\ln x)$$

$$\Rightarrow\frac{dy}{dx}=y\left[\frac{1+(\ln x)\ \ln(\ln x)}{(\ln x)}\right] \qquad \text{from (i)}$$

$$\frac{dy}{dx}=(\ln x)^x\left[\frac{1+(\ln x)\ln(\ln x)}{(\ln x)^1}\right]$$

$$\Rightarrow \frac{d}{dx}(\ln x)^x = (\ln x)^{x-1}\left[1+(\ln x)\ln(\ln x)\right]$$

57. **Find** $f'(x)$ **if** $f(x) = \dfrac{e^x}{e^{-x}+1}$ **(C.W)**

Sol: $f'(x) = ?$

Given $f(x) = \dfrac{e^x}{e^{-x}+1}$ Diff. w.r.t. x.

$$\frac{d}{dx}f(x) = \frac{d}{dx}\left(\frac{e^x}{e^{-x}+1}\right)$$

$$f'(x) = \frac{(e^{-x}+1)\frac{d}{dx}e^x - e^x\frac{d}{dx}(e^{-x}+1)}{(e^{-x}+1)^2}$$

$$f'(x) = \frac{(e^{-x}+1)e^x - e^x(-e^{-x})}{(e^{-x}+1)^2}$$

$$f'(x) = \frac{e^{-x}.e^x + e^x + e^x.e^{-x}}{(e^{-x}+1)^2}$$

$$f'(x) = \frac{e^{-x+x} + e^x + e^{x-x}}{(e^{-x}+1)^2} = \frac{e^0 + e^x + e^0}{(e^{-x}+1)^2}$$

$$f'(x) = \frac{1+e^x+1}{(e^{-x}+1)^2} = \frac{2+e^x}{(e^{-x}+1)^2}$$

58. **Find** $\dfrac{dy}{dx}$ **if** $y = (x^2+5)(x^3+7)$ **(C.W)**

Sol: $\dfrac{dy}{dx} = ?$

Given $y = (x^2+5)(x^3+7)$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left[(x^2+5)(x^3+7)\right]$$

Using product Rule

$$\frac{dy}{dx} = (x^2+5)\frac{d}{dx}(x^3+7) + (x^3+7)\frac{d}{dx}(x^2+5)$$

$$\frac{dy}{dx} = (x^2+5)(3x^2) + (x^3+7)(2x)$$

$$\frac{dy}{dx} = 3x^4 + 15x^2 + 2x^4 + 14x$$

$$\frac{dy}{dx} = 5x^4 + 15x^2 + 14x$$

59. **Find** $\frac{dy}{dx}$ **if** $y=\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$ **(C.W)**

Sol: $\frac{dy}{dx}=?$

Given $y=\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$

$y=\left(\sqrt{x}\right)^2+\left(\frac{1}{\sqrt{x}}\right)^2-2\left(\sqrt{x}\right)\left(\frac{1}{\sqrt{x}}\right)$

$y=x+\frac{1}{x}-2$

$y=x+x^{-1}-2$

Diff. w.r.t. x.

$\frac{dy}{dx}=\frac{d}{dx}x+\frac{d}{dx}x^{-1}-\frac{d}{dx}2$

$\frac{dy}{dx}=1+(-1)x^{-1-1}-0$

$\frac{dy}{dx}=1-\frac{1}{x^2}=\frac{x^2-1}{x^2}$

60. **Find** $\frac{dy}{dx}$ **if** $y=\frac{x^2+1}{x^2-3}$ **(H.W)**

Sol: $\frac{dy}{dx}=?$

Given $y=\frac{x^2+1}{x^2-3}$ diff. w.r.t. x.

$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^2+1}{x^2-3}\right)$ using Quotient Rules

$$\frac{dy}{dx}=\frac{\left(x^2-3\right)\frac{d}{dx}\left(x^2+1\right)-\left(x^2+1\right)\frac{d}{dx}\left(x^2-3\right)}{\left(x^2-3\right)^2}$$

$$\frac{dy}{dx}=\frac{\left(x^2-3\right)(2x)-\left(x^2+1\right)(2x)}{\left(x^2-3\right)^2}$$

$$\frac{dy}{dx}=\frac{(2x)\left[\left(x^2-3\right)-\left(x^2+1\right)\right]}{\left(x^2-3\right)^2}$$

$$\frac{dy}{dx}=\frac{2x\left(\cancel{x^2}-3-\cancel{x^2}-1\right)}{\left(x^2-3\right)^2}$$

$$\frac{dy}{dx}=\frac{(2x)(-4)}{\left(x^2-3\right)^2}=\frac{8x}{\left(x^2-3\right)^2}$$

LONG QUESTION'S OF CHAPTER-2 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Derivative of a function by definition:

1. Differentiate ab-intio with respect to x if $y = \sin\sqrt{x}$ **(C.W)**
2. Differentiate $\cos\sqrt{x}$ w.r.t x from first principle. **(C.W)**

Topic II: Direct Differentiation:

3 If $y = x^4 + 2x^2 + 2$ then prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$ **(H.W)**

Topic III: Chain Rule:

4. Differentiate $x^2 - \frac{1}{x^2}$ w.r.t. x^4 **(H.W)**

5. If $x = a\cos^3\theta, y = b\sin^3\theta$, show that $\frac{dy}{dx} + b\tan\theta = 0$ **(H.W)**

6. Find $\frac{dy}{dx}$ if x = a (cos t + sin t), y = a (sin t – t cos t). **(H.W)**

7. Prove that $y\frac{dy}{dx} + x = 0$ if $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$ **(H.W) (2 times)**

8. Find $\frac{dy}{dx}$ If $x = \theta + \frac{1}{\theta}$, $y = \theta + 1$ **(C.W)**

Topic IV: Derivative of Tringnometric Functions:

9. Differentiate with respect to x if $\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$ **(C.W)**

10. Show that $\frac{dy}{dx} = \frac{y}{x}$ If $\frac{y}{x} = Tan^{-1}(\frac{x}{y})$ **(H.W) (4 times)**

11. If $y = \tan(p\tan^{-1}x)$, show that $(1 + x^2)y_1 - p(1 + y^2) = 0$. **(C.W)**

12. If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \dots\infty}}}$ then prove that $(2y-1)\frac{dy}{dx} = \sec^2 x$. **(C.W)**

Topic V: Derivative of exponential and Logrithmic Function:

13 If $y = e^{ax}\,\text{Sin}bx$ then show that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx}(a^2 + b^2)y = 0$ **(H.W)**

14 Find f′ (x), when $f(x) = (\ell nx)^{\ell nx}$ **(H.W)**

Topic VI: Higher Derivative:

15. If $x = a(\theta + \sin)$ and $y = (1 + \cos\theta)$, then shows that $y^2\frac{d^2y}{dx^2} + a = 0$ **(C.W)**

16. If $y = e^x\sin x$, show that $\frac{d^2y}{dx^2} - 2y\frac{dy}{dx} + 2y = 0$ **(C.W)**

17. If y = a cos (ln x) + b sin (ln x), then prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ **(H.W)**

18 If $y = (\text{Cos}^{-1}x)^2$ prove that $(1-x^2)y_2 - xy_1 - 2 = 0$ **(C.W) (2 times)**

Topic VIII: Increasing and Decreasing Functions:

19. Show that $y = x^x$ has minimum value at x = 1 /e **(H.W)**

20. Show that $y = \frac{\ell nx}{x}$ has maximum value at x = e **(C.W)**

Chapter-2 (Examples According to ALP Smart Syllabus)

Example 2: (Pag#46) Find the derivative of $\sqrt{x}$ at x = a from first principles. (C.W)

Sol: If f(x) = $\sqrt{x}$, then

(i) $f(x+\delta x)=\sqrt{x+\delta x}$ and

(ii) $f(x+\delta x)-f(x)=\sqrt{x+\delta x}-\sqrt{x}$

$$f(x+\delta x)-f(x)=\frac{\left(\sqrt{x+\delta x}-\sqrt{x}\right)\left(\sqrt{x+\delta x}+\sqrt{x}\right)}{\sqrt{x+\delta x}+\sqrt{x}}=\frac{(x+\delta x)-x}{\sqrt{x+\delta x}+\sqrt{x}}$$

i.e., $f(x+\delta x)-f(x)=\dfrac{\delta x}{\sqrt{x+\delta x}+\sqrt{x}}$ $\qquad (I)$

(iii) Dividing both sides of (I) by $\delta x \to 0$, we have

$$\lim_{\delta x\to 0}\frac{f(x+\delta x)-f(x)}{\delta x}=\lim_{\delta x\to 0}\frac{\delta x}{\sqrt{x+\delta x}+\sqrt{x}}=\frac{1}{\sqrt{x+\delta x}+\sqrt{x}},(\because \delta x\neq 0)$$

(iv) Taking limit of both the sides as $\delta x \to 0$, we have

$$\frac{f(x+\delta x)-f(x)}{\delta x}=\left(\frac{1}{\sqrt{x+\delta x}+\sqrt{x}}\right)$$

i.e., f′(x) = $\dfrac{1}{\sqrt{x}+\sqrt{x}}=\dfrac{1}{2\sqrt{x}},(x>0)$

and f′(a) = $\dfrac{1}{2\sqrt{a}}$

or Putting x = a in f(x) = $\sqrt{x}$, gives f(a) = $\sqrt{a}$

So, f(x) - f(a) = $\sqrt{x}$ - $\sqrt{a}$

Using alternative form for the definition of a derivative, we have

$$\frac{f(x)-f(a)}{x-a}=\frac{\sqrt{x}-\sqrt{a}}{x-a}$$

$$=\frac{\left(\sqrt{x}-\sqrt{a}\right)\left(\sqrt{x}+\sqrt{a}\right)}{(x-a)\left(\sqrt{x}+\sqrt{a}\right)}$$ (rationalizing the numerator)

$$=\frac{x-a}{(x-a)\left(\sqrt{x}+\sqrt{a}\right)}=\frac{1}{\sqrt{x}+\sqrt{a}},\ (x\neq a) \qquad (II)$$

Taking limit of both the sides of (II) as $x \to a$, gives

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\lim_{x\to a}\frac{1}{\sqrt{x}+\sqrt{a}}=\frac{1}{\sqrt{a}+\sqrt{a}}$$

i.e. $f'(a)=\dfrac{1}{2\sqrt{a}}$

Example 7: (Page#59) Differentiate $\dfrac{(\sqrt{x}+1)(x^{2/3}-1)}{x^{3/2}-x^{1/2}}$ **with respect to x.**

Sol: Let $y=\dfrac{(\sqrt{x}+1)(x^{2/3}-1)}{x^{3/2}-x^{1/2}}$

$$y=\frac{(\sqrt{x}+1)\left((\sqrt{x})^3-1\right)}{\sqrt{x}(x-1)}$$

$$y=\frac{(\sqrt{x}+1)(\sqrt{x}-1)(x+\sqrt{x}+1)}{\sqrt{x}(x-1)}=\frac{(x-1)(x+\sqrt{x}+1)}{\sqrt{x}(x-1)}=\frac{x+\sqrt{x}+1}{\sqrt{x}}$$

Differentiating with respect to x, we have

$$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{x+\sqrt{x}+1}{\sqrt{x}}\right]=\frac{\sqrt{x}\frac{d}{dx}(x+\sqrt{x}+1)-(x+\sqrt{x}+1)\frac{d}{dx}(\sqrt{x})}{(\sqrt{x})^2}$$

$$\frac{dy}{dx}=\frac{\sqrt{x}\left(1+\frac{1}{2}x^{-\frac{1}{2}}+0\right)-(x+\sqrt{x}+1)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{x}$$

$$\frac{dy}{dx}=\frac{\sqrt{x}\left(1+\frac{1}{2\sqrt{x}}\right)-(x+\sqrt{x}+1)\frac{1}{2\sqrt{x}}}{x}$$

$$\frac{dy}{dx}=\frac{\sqrt{x}\left(1+\frac{1}{2\sqrt{x}}\right)-\frac{(x+\sqrt{x}+1)}{2\sqrt{x}}}{x}=\frac{2x+\sqrt{x}-x-\sqrt{x}-1}{\sqrt{x}.2\sqrt{x}}=\frac{x-1}{2x^{3/2}}$$

Example 8: Differentiate $\dfrac{2x^3-3x^2+5}{x^2+1}$ **with respect to x.**

Sol: Let $\phi(x)=\dfrac{2x^3-3x^2+5}{x^2+1}$ Then we take

$$f(x)=2x^3-3x^2+5 \ \ and \ \ g(x)=x^2+1$$

Now $f'(x)=\dfrac{d}{dx}\left[2x^3-3x^2+5\right]=2(3x^2)-3(2x)+0=6x^2-6x$

$and \ g'(x)=\dfrac{d}{dx}\left[x^2+1\right]=2x+0=2x$

Using the quotient formula $\phi'(x)=\dfrac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$ **we obtain**

$$\frac{d}{dx}=\left[\frac{2x^3-3x^2+5}{x^2+1}\right]=\frac{(6x^2-6x)(x^2+1)-(2x^3-3x^2+5)(2x)}{(x^2+1)^2}$$

$$=\frac{6x^4-6x^3+6x^2-6x-(4x^4-6x^3+10x)}{(x^2+1)^2}$$

$$=\frac{6x^4-6x^3+6x^2-6x-4x^4+6x^3-10x}{(x^2+1)^2}=\frac{2x^4+6x^2-16x}{(x^2+1)^2}$$

Example 2(ii): Differentiate ab-initio w.r.t. 'x' $\sin\sqrt{x}$

Sol: Let $y=\sin\sqrt{x}$, then $y+\delta y=\sin\sqrt{x+\delta x}$,

And $\delta y=\sin\sqrt{x+\delta x}-\sin\sqrt{x}$

$$=2\cos\left(\frac{\sqrt{x+\delta x}+\sqrt{x}}{2}\right)\sin\left(\frac{\sqrt{x+\delta}-\sqrt{x}}{2}\right)$$

As $\left(\sqrt{x+\delta x}+\sqrt{x}\right)\left(\sqrt{x+\delta x}-\sqrt{x}\right)=(x+\delta x)-x=\delta x$

So $$\frac{\delta y}{\delta x}=2\cos\left(\frac{\sqrt{x+\delta x}+\sqrt{x}}{2}\right).\frac{\sin\left(\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}\right)}{\delta x}$$

$$\frac{\delta y}{\delta x}=\frac{2\cos\left(\frac{\sqrt{x+\delta x}+\sqrt{x}}{2}\right).\sin\left(\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}\right)}{\left(\sqrt{x+\delta x}+\sqrt{x}\right)\left(\sqrt{x+\delta x}-\sqrt{x}\right)}$$

$$=\frac{\cos\left(\frac{\sqrt{x+\delta x}+\sqrt{x}}{2}\right)}{\left(\sqrt{x+\delta x}+\sqrt{x}\right)}.\frac{\sin\left(\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}\right)}{\left(\sqrt{x+\delta x}-\sqrt{x}\right)}$$

$$Thus\ \frac{dy}{dx}=\lim_{\delta x\to 0}\left(\frac{\cos\left(\frac{\sqrt{x+\delta x}+\sqrt{x}}{2}\right)}{\left(\sqrt{x+\delta x}+\sqrt{x}\right)}\right).\lim\left(\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}\right)\to 0\left[\frac{\sin\left(\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}\right)}{\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}}\right]$$

$$\frac{dy}{dx}=\left(\frac{\cos\left(\frac{\sqrt{x}+\sqrt{x}}{2}\right)}{\left(\sqrt{x}+\sqrt{x}\right)}\right).1=\frac{\cos\sqrt{x}}{2\sqrt{x}}\quad\left(\because \frac{\sqrt{x+\delta x}-\sqrt{x}}{2}\to 0\ when\ \delta x\to 0\right)$$

Example 1: (Pag#83) Find if $\frac{dy}{dx}$ $y=\log_{10}\left(ax^2+bx+c\right)$ **(C.W)**

Sol: Let $u=ax^2+bx+c$ then

$$y=\log_{10}^{u}\Rightarrow\frac{dy}{du}=\frac{1}{u}.\frac{1}{\ln 10}$$

And $\frac{du}{dx}=\frac{d}{dx}.\left(ax^2+bx+c\right)=a(2x)+b(1)=2ax+b$

Thus $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}=\left(\frac{1}{u}.\frac{1}{\ln 10}\right)\frac{du}{dx}=\frac{1}{\left(ax^2+bx+c\right)\ln 10}(2ax+b)$

Or $\frac{d}{dx}.\left[\log_{10}\left(ax^2+bx+c\right)\right]=\frac{2ax+b}{\left(ax^2+bx+c\right)\ln 10}$

Example 3: (Pag#84) Differentiate (ln x)^x w.r.t 'x' **(C.W)**

Sol: Let y = (ln x)^x

Taking logarithm of both sides of (i) we have

$$\ln y=\ln\left[(\ln x)^x\right]=x\ln(\ln x)$$

Differentiate w.r.t 'x'.

$$\frac{1}{y}\frac{dy}{dx}=1.\ln(\ln x)+x.\frac{1}{\ln x}.\frac{d}{dx}(\ln x)$$

$$=\ln(\ln x)+x.\frac{1}{\ln x}.\frac{1}{x}=\ln(\ln x)+\frac{1}{\ln x}$$

$$\frac{dy}{dx}=y\left[\ln(\ln x)+\frac{1}{\ln x}\right]=(\ln x)^x\left[\ln(\ln x)+\frac{1}{\ln x}\right]$$

Example 7: (Pag#94) If $y=\sin^{-1}\frac{x}{a}$ then show that $y_2=x\left(a^2-x^2\right)^{-\frac{3}{2}}$ **(C.W)**

Sol: $y=\sin^{-1}\frac{x}{a}$ so

$$y_1=\frac{dy}{dx}=\frac{d}{dx}\left[\sin^{-1}\frac{x}{a}\right]=\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\times\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$=\frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}}.\frac{1}{a}=\frac{a}{\sqrt{a^2-x^2}}.\frac{1}{a}=\left(a^2-x^2\right)^{-1/2}$$

$$y_2=\frac{d}{dx}\left[\left(a^2-x^2\right)^{-1/2}\right]=-\frac{1}{2}\left(a^2-x^2\right)^{-3/2}\times(-2x)=x\left(a^2-x^2\right)^{-3/2}$$

Example 1: Expand $f(x)=\dfrac{1}{1+x}$ in the Maclaurin series.

Sol: f is defined at x = 0 that is, f(0) = 1. Now we find successive derivatives of f and their values at x = 0.

$f'(x) = (-1)(1+x)^{-2}$ and $f'(0) = -1$

$f''(x) = (-1)(-2)(1+x)^{-3}$ and $f''(0) = (-1)^2 \lfloor 2$

$f'''(x) = (-1)(-2)(-3)(1+x)^{-4}$ and $f'''(0) = (-1)^3 \lfloor 3$

$f^{(4)}(x) = (-1)(-2)(-3)(-4)(1+x)^{-5}$ and $f^{(4)}(x) = (-1)^4 \lfloor 4$

Following the pattern, we can write $f^{(n)}(0) = (-1)^n \lfloor n$

Now substituting $f(0) = 1$, $f'(0) = -1$, $f''(0) = (-1)^2 \lfloor 2$

$f'''(0) = (-1)^3 \lfloor 3$, $f^{(4)}(0) = (-1)^4 \lfloor 4$,... $f^{(n)}(0) = (-1)^n \lfloor n$ in the formula.

$$f(x)=f(0)+f'(0)x+\frac{f''(0)}{\lfloor 2}x^2+\frac{f'''(0)}{\lfloor 3}x^3+\frac{f^{(4)}(0)}{\lfloor 4}x^4+\ldots+\frac{f^{(n)}(0)}{\lfloor n}x^n+\ldots$$

We have

$$\frac{1}{1+x}=1+(-1)x+(-1)^2\frac{\lfloor 2}{\lfloor 2}x^2+(-1)^3\frac{\lfloor 3}{\lfloor 3}x^3+(-1)^4\frac{\lfloor 4}{\lfloor 4}x^4+\ldots+\frac{(-1)^n\lfloor n}{\lfloor n}x^n+\ldots$$

$$=1-x+x^2-x^3+x^4+\ldots+(-1)^n x^n+\ldots$$

Thus, the Maclaurin series for $\dfrac{1}{1+x}$ is the geometric series with the first term 1 and common ratio $-x$.

Example 2: Examine the function defined $f(x) = 1 + x^3$ for extreme values.

Sol: Given that $f(x) = 1 + x^3$

Differentiating w.r.t 'x', we get $f'(x) = 3x^2$

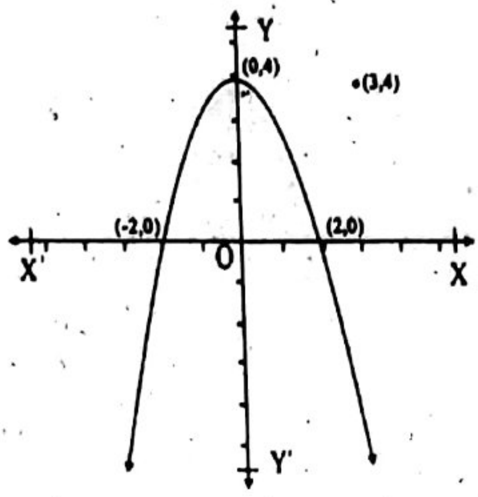
$$f'(x)=0 \Rightarrow 3x^2=0 \Rightarrow x=0$$
$$f'(x)=6x \text{ and } f''(0)=6(0)=0$$

The second derivative does not help in determining the extreme values.

$$f'(0-\varepsilon)=3(0-\varepsilon)^2=3\varepsilon^2>0$$
$$f'(0+\varepsilon)=3(0+\varepsilon)^2=3\varepsilon^2>0$$

As the first derivative does not change sign at x = 0, therefore (0,0) is a point of inflexion.

Example 5: Find the point on the graph of the curve $y = 4 - x^2$ which is closest to the point (3,4)



Sol: Let be distance between a point (x,y) on the curve y = 4 – x^2 and the point (3,4), Then

$$l=\sqrt{(x-3)^2+(y-4)^2}=\sqrt{(x-3)^2+(4-x^2-4)}\ \left(\because (x,y)\, is\, on\, the\, curve\, y=4-x^2\right)$$

$$=\sqrt{(x-3)^2+x^4}$$

Now we find x for which l is minimum.

$$\frac{dl}{dx}=\frac{1}{2.\sqrt{(x-3)^2+x^4}}\left[2(x-3)+4x^3\right]$$

$$=\frac{1}{2l}.2\left(2x^3+x-3\right)=\frac{1}{l}\left(2x^3+x-3\right)=\frac{1}{l}(x-1)\left(2x^2+2x+3\right)$$

$$\frac{dl}{dx}=0\Rightarrow\frac{1}{l}(x-1)\left(2x^2+2x+3\right)=0\Rightarrow x-1=0\ or\ 2x^2+2x+3=0$$

$$\Rightarrow x=1\qquad\left(\because 2x^2+2x+3=0\, gives\, comples\, roots\right)$$

l is positive for $1-\varepsilon$ and $1+\varepsilon$ where ε is very very small positive real number.

Also $2x^2+2x+3=2\left(x^2+x+\frac{1}{4}\right)+\frac{5}{2}=2\left(x+\frac{1}{2}\right)^2+\frac{5}{2}$ is positive, for x = $1-\varepsilon$ and x = $1+\varepsilon$

The sign of $\frac{dl}{dx}$ depends on the factor $x-1$

x - 1 is negative for x = $1-\varepsilon$ because x – 1 = $1-\varepsilon$ - 1 = $-\varepsilon$ (i)

x - 1 is negative for x = $1-\varepsilon$ because x – 1 = $1-\varepsilon$ - 1 = ε (ii)

From (i) and (ii) we conclude that $\frac{dl}{dx}$ changes sign from –ve to +ve at x = 1 Thus l has a minimum value at x = 1

Putting x = 1 in y = 4 – x^2 , we get the y-coordinate of the required point which is 4 – $(1)^2$ = 3

Hence the required point on the curve is (1,3).

OBJECTIVES (MCQ'S) OF CHAPTER-3 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Differential of Variables:

1. If y = f(x) is a differential function then differential of X is defined by relation

(A) $dx = \delta y$ (B) dx = dy (C) $\delta x = dy$ (D) $dx = \delta x$

2. Differential of y is denoted by. **(2 times)**

(a) dy' (b) $\frac{dy}{dx}$ (c) dy (d) dx

3. If v = x^3, then differential of v is. **(2 Times)**

(a) $3x^3$ (b) $3x^2$ dv (c) x^3 dv (d) $3x^2$ dx

4. $f(x+\delta x) \approx$

(A) f′ (x) dx (B) f(x) –f′ (x) dx (C) f (x) + f′ (x) dx (D) –f′ (x) dx

Topic II: Anti-Derivative (Integration):

5. If $\phi'(x) = f(x)$, then $\phi(x)$ is called ________ of f(x) **(2 times)**

(A) Derivative (B) Integral (C) Differential coefficient (D) Area

6. $\int \sec^2 nx dx =$ **(4 times)**

(A) $\frac{n}{3}\sec 3nx + c$ (B) $n\tan nx + c$ (C) $\tan nx + c$ (D) $\frac{1}{n}\tan nx + c$

7. If α is constant, then $\int \cot\alpha dy =$ **(2 times)**

(A) $\sin\alpha + c$ (B) $-\sin\alpha + c$ (C) $y\cot\alpha + c$ (D) $x\sin\alpha + c$

8. $\int sex5x\tan 5x dx$ **is equal to:** **(3 times)**

(A) 5 sec 5x + c (B) $\frac{\sec x}{5} + c$ (C) $\frac{\sec 5x}{5} + c$ (D) $\frac{\tan 5x}{5} + c$

9. $\int \cos 2x dx$ **is equal to:** **(1 time)**

(A) – 2sin2x + c (B) 2sin2x+ c (C) $\frac{-\sin 2x}{2} + c$ (D) $\frac{\sin 2x}{2} + c$

10. $\int (ax+b)^n dx$ **is equal to:** **(3 times)**

(A) $\frac{(ax+b)^{n+1}}{(n+1)a} + c$ (B) $\frac{(ax+b)^{n-1}}{n-1} + c$ (C) $\frac{(ax+b)^{n+1}}{n+1} + c$ (D) $\frac{(ax+b)^n}{n+1} + c$

11. $\int \frac{1}{ax-1} dx =$ ________ **(2 times)**

(A) ln (ax – 1) + c (B) a ln (ax – 1) + c (C) $\frac{1}{a}\ln(ax-1) + c$ (D) $\frac{-1}{(ax-1)^2}$

12. $\int e^{ax} dx =$ ________ **(3 times)**

(A) $e^{ax} + c$ (B) $e^a + c$ (C) $ae^{ax} + c$ (D) $\frac{1}{a}e^{ax} + c$

13. $\int \cot x dx$ **is equals:** **(6 times)**

(A) $-Co\sec^2 x + c$ (B) $Co\sec^2 x + c$ (C) $\ell n Cosx + c$ (D) $\ln|\sin x| + c$

14. $\int e^x dx$ **is equal to:** **(5 times)**

(A) $xe^x + c$ (B) $xe^{x-1} + c$ (C) $e^{x-1} + c$ (D) $e^x + c$

15. $\int e^{\lambda x+\mu}dx =$ ________ **(3 times)**

(A) $\frac{1}{\lambda}e^{\lambda x+\mu}+c$ (B) $\frac{1}{\mu}e^{\lambda x+\mu}+c$ (C) $\lambda e^{\lambda x+\mu}+c$ (D) $\mu e^{\lambda x+\mu}+c$

16. For $n \neq -1$, $\int x^n dx =$ ________ **(3 times)**

(A) $\frac{x^{n-1}}{n-1}$ (B) $x^{n+1}+c$ (C) $\frac{x^{n+1}}{n+1}+c$ (D) $\frac{x^n}{n+1}+c$

17. $\int \frac{1}{\sqrt{1-x^2}}$ **dx is equal to.** **(3 Times)**

(a) $\tan^{-1}x$ (b) $\cot^{-1}x$ (c) $\cos^{-1}x$ (d) $\sin^{-1}x$

18. $\int -\sin x\, dx$ **is equal to.**

(a) sin x+c (b) cos x+c (c) – cos x+c (d) – sin x+c

19. $\int (3x^2+2x)$ **dx is equal to.**

(a) 6x + 2 (b) $x^3 + x^2$ (c) 3x + 2 (d) $\frac{x^3}{3}+\frac{x^2}{2}$

20. $\int (e^x+1)$ **dx is equal to.** **(2 Times)**

(a) e^x (b) $e^x + x$ (c) $e^x - x$ (d) $e^x + x^2$

21. $\int \tan x$ **dx =** **(3 Times)**

(a) $\ell n \sec x$ (b) $\sec^2 x$ (c) $\ell n \cos x$ (d) $\ell n \sin x$

22. $\int (2x+3)^{\frac{1}{2}} dx$, *equals* :

(a) $(2x+3)^{\frac{1}{2}}+c$ (b) $(2x+3)^{\frac{3}{2}}+c$ (c) $\frac{1}{3}(2x+3)^{\frac{1}{2}}+c$ (d) $\frac{1}{3}(2x+3)^{\frac{3}{2}}+c$

23. **Anti derivative of cot x, equal.** **(3 Times)**

(a) $\ln|\cos x|+c$ (b) $\ln|\sin x|+c$ (c) $-\operatorname{cosec}^2 x+c$ (d) $\ln|\sec x|+c$

24. $\int \sin 2x$ dx =

(a) $-\frac{Cos2x}{2}$ (b) $\frac{Cos2x}{2}$ (c) $-2Sin2x$ (d) $-2Cos2x$

25. $\int \tan\frac{\pi}{4}dx =$

(a) $\ell n(\sin\frac{\pi}{4})$ (b) $\frac{1}{4}\sec^2\frac{\pi}{4}$ (c) $\sec^2\frac{\pi}{4})$ (d) $x\tan\frac{\pi}{4}$

26. $\int \frac{\sin p}{\cos^2 x}dx =$

(a) $\text{Sinpsec}^2 x$ (b) sinptan x (c) $\text{cospsec}^2 x$ (d) $\sec^2 x$

27. $\int \frac{1}{x}$ **dx = :** **(4 Times)**

(A) $\frac{1}{x^2}$ (B) $-\frac{1}{x^2}$ (C) $\frac{1}{x}$ (D) lnx

28. $\int a^x$ **dx = :** **(4 Times)**

(A) $\frac{a^x}{\ell na}$ (B) a^x (C) $\frac{\ell na}{a^x}$ (D) $a^x\, \ell na$

29. $\int \frac{1}{1+\cos x}$ **dx is equal to** **(2 times)**

(A) $\tan\frac{x}{2}$ (B) $\frac{1}{2}\tan\frac{x}{2}$ (C) $\cot\frac{x}{2}$ (D) $\frac{1}{2}\cot\frac{x}{2}$

30. $\int \frac{-1}{1+x^2}dx$ **equals**

(A) $-\tan^{-1}x$ (B) $\tan x^2$ (C) $\cot^{-1}x^2$ (D) $\cot^{-1}x$

31. $\int \sec x\, dx$ **equals:-**

(A) $\sec x \tan x$ (B) $\ell n(\sec x \tan x)$ (C) $\ell n(\sec x + \tan x)$ (D) $\ell n(\sec x - \tan x)$

32. $\int \frac{1}{1+x^2}dx =$ ________ **(2 Times)**

(A) $\tan^{-1}x$ (B) $\cot^{-1}x$ (C) $\cos^{-1}x$ (D) $\sin^{-1}x$

33. $\int x(\sqrt{x}+1)dx$ **is equal to:**

(A) $\frac{2}{3}x^{3/2}+c$ (B) $\frac{2}{5}x^{5/2}+\frac{x^2}{2}+c$ (C) $\frac{2}{5}x^{5/2}+c$ (D) $x^{3/2}+x+c$

34. $\int \sin x\, dx$ **is equal to:**

(A) $\cos x$ (B) $\sin x$ (C) $-\sin x$ (D) $-\cos x$

35. **The integration is the reverse process of:**

(A) Induction (B) Differentiation (C) Tabulation (D) Sublimation

36. $\int e^{ax}dx =$

(A) e^{ax} (B) ae^{ax} (C) xe^{ax} (D) $\frac{e^{ax}}{a}$

37. $\int a^x \ln a\, dx =$

(A) $a^x + c$ (B) $\frac{a^x}{\ln a}$ (C) $\ln a^x + c$ (D) $\ln a\, a^x + c$

38. $\int 0\, dx =$

(A) 1 (B) 0 (C) Constant (D) x

39. $\int \frac{x}{x+2}dx =$:

(A) $\ell n(x+2)+c$ (B) $x+2\ell n(x+2)+c$ (C) $x-2\ell n(x+2)+c$ (D) $x-\ell n(x+2)+c$

40. $\int \tan x\, dx =$: **(3 Times)**

(A) $\ell n \cot x + c$ (B) $\ell n \cos x + c$ (C) $\ell n \sin x + c$ (D) $\ell n \sec x + c$

41. $\int (2x)^{3/2}dx =$

(A) $\frac{1}{5}(2x)^{5/2}+C$ (B) $\frac{2}{5}(2x)^{5/2}+C$ (C) $\frac{1}{2}(2x)^{3/2}+C$ (D) $\frac{2}{3}(2x)^{1/2}+C$

42. $\int 3^{\lambda x}dx =$

(A) $\frac{3^{\lambda x}}{\lambda \ln 3}+c$ (B) $\frac{3^{\lambda x}}{\lambda}\ln 3 + c$ (C) $\frac{\lambda 3^{\lambda x}}{\ln 3}+c$ (D) $\frac{3^{\lambda x}}{\ln 3}+c$

43. $\int \tan^2 x\, dx$ **is equal to:**

(A) $2\tan x$ (B) $2\tan x + x$ (C) $\tan x + x$ (D) $\tan x - x$

44. $\int \sin 2x\, dx =$

(A) $\frac{-\cos 2x}{2}$ (B) $\frac{\cos 2x}{2}$ (C) $2\cos 2x$ (D) $-2\cos 2x$

Topic III: Substitution Method:

45. $\int \frac{1}{(1+x^2)Tan^{-1}x}dx =$ **(3 times)**

(A) $ln|1+x^2|+c$ (B) $(1+x^2)^2+c$ (C) $ln|Tan^{-1}x|+c$ (D) $Tan^{-1}x+c$

46. $\int \frac{2x}{\sqrt{1-x^2}}dx =$ ________ **(3 times)**

(A) $\ln|1-x^2|+c$ (B) $-2\sqrt{1-x^2}+c$ (C) $2\sqrt{1-x^2}+c$ (D) $\sqrt{1-x^2}+c$

47. $\int \frac{1}{x\ln x}dx =$ ____________ **(5 times)**

(A) $\frac{1}{x}+c$ (B) lnx + c (C) $(\ln x)^2+c$ (D) ln (lnx) + c

48. **The anti-derivative of $\frac{1}{(1+x^2).\tan^{-1}x}$ equals:** **(2 times)**

(A) $\ell n(\tan^{-1}x)+c$ (B) $\ell n(\tan x)+c$ (C) $\tan^{-1}x+c$ (D) $\tan x+c$

49. $\int \frac{f'(x)}{f(x)}dx$ **is equal to:** **(3 times)**

(A) $\ell nf'(x)+c$ (B) $\ell nf(x)+c$ (C) $\ell nf^n(x)+c$ (D) $f(x)+c$

50. $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx =$ ____________ **(3 times)**

(A) $e^{\sec x}+c$ (B) $e^{\tan x}+c$ (C) $e^{\cot^{-1}x}+c$ (D) $e^{\tan^{-1}x}+c$

51. **If the expressions involves $\sqrt{x^2-a^2}$, then the suitable substitution is :**

(A) $x=a\sin\theta$ (B) $x=a\sec\theta$ (C) $x=a\cos\theta$ (D) $x=\sin\theta$

52. $\int \operatorname{cosec}^2 2x dx =$

(a) cos 2 x (b) $\frac{-1}{2}\cot 2x$ (c) 2 tan 2 x (d) $\frac{1}{2}\cot 2x$

53. $\int \frac{\sec^2 x}{\tan x}dx =$

(a) tan x (b) $\ell n\cot x$ (c) cot x (d) $\ell n\tan x$

54. $\int \frac{\ell nx}{x}dx =$ **(4 Times)**

(a) x (b) $\frac{(\ell nx)^2}{2}$ (c) $\frac{1}{\ell nx}$ (d) $\ell nx(\ell nx)$

55. $\int \sec^2 2x$ **dx**

(a) $\frac{1}{2}\tan 2x$ (b) $\tan 2x$ (c) $\frac{1}{2}\tan x$ (d) $2\tan 2x$

56. $\int e^{\sin x}\cos x$ **dx** **(4 Times)**

(a) ln sin x + C (b) ln cos x + C (c) $e^{\cos x}+C$ (d) $e^{\sin x}+C$

57. $\int -Co\sec^2 2x$ dx =

(a) $\frac{\cos 2x}{2}$ (b) $\frac{Cot2x}{2}$ (c) $-\frac{Cot2x}{2}$ (d) $Cot2x$

58. $\int f^n(x).f'(x)$ **dx where n ≠ - 1 equals.**

(a) $n f^{n-1}$ (b) $n f^{n+1}$ (c) $\frac{f^{n+1}(x)}{n+1}$ (d) $\frac{f^n(x)}{n+1}$

59. $\int \frac{1}{\sqrt{a^2-x^2}}dx$ **dx is equal to** **(2 times)**

(a) $Sin^{-1}(\frac{a}{x})$ (b) $Cos^{-1}(\frac{a}{x})$ (c) $Cos^{-1}(\frac{x}{a})$ (d) $Sin^{-1}(\frac{x}{a})$

60. $\int e^{\tan x}Sec^2 dx =$

(a) $e^{\tan x}$ (b) $e^{-\tan x}$ (c) $e^{\cot x}$ (d) $e^{-\cot x}$

61. $\int Cot^3x(-Co\sec^2 x)dx$

(a) $\frac{Cot^3x}{3}$ (b) $\frac{-Cot^3x}{3}$ (c) $\frac{-Cot^4x}{4}$ (d) $\frac{Cot^4x}{4}$

62. $\int\sqrt{2x+3}(2dx)=$

(a) $\frac{2}{3}(2x+3)^{\frac{3}{2}}$ (b) $\frac{3}{2}(2x+3)^{\frac{3}{2}}$ (c) $\frac{-2}{3}(2x+3)^{\frac{3}{2}}$ (d) $\frac{-3}{2}(2x+3)^{\frac{3}{2}}$

63. $\int\frac{\sin 2x}{\sin x}dx=$

(a) Sin2x (b) 2 sin2x (c) $\frac{1}{2}\sin x$ (d) 2sinx

64. $\int \sin^3x \cos x\, dx =$

(A) $\frac{sin^3x}{3}$ (B) $\frac{sin^4x}{4}$ (C) $\frac{sin^5x}{5}$ (D) $\frac{cos^{-4}x}{-4}$

65. $\int \frac{e^{Sec^{-1}x}}{x\sqrt{x^2-1}}$ dx is:

(A) $e^{Sec^{-1}x}$ (B) $e^{Co\,Sec^{-1}x}$ (C) $e^{Tan^{-1}x}$ (D) $e^{Cot^{-1}x}$

66. $\int \frac{dx}{\sqrt{5-x^2}}$ = :

(A) $\sin^{-1}\frac{5}{x}$ (B) $\sin^{-1}\frac{x}{5}$ (C) $\sin^{-1}\frac{\sqrt{5}}{x}$ (D) $\sin^{-1}\frac{x}{\sqrt{5}}$

67. $\int\frac{dx}{ax+b}$

(A) ln (ax + b) (B) aln (ax + b) (C) $\frac{1}{a}$ ln (ax + b) (D) $\log_a$ (ax + b)

68. $\int\frac{1}{x^2+16}$ dx

(A) $\tan\frac{x}{4}$ (B) $\frac{1}{4}\tan^{-1}\frac{x}{4}$ (C) $\frac{1}{4}\tan\frac{x}{4}$ (D) $\tan^{-1}\frac{x}{4}$

69. $\int$ Sec² x tan x dx = **(2 times)**

(A) Sec x tan² x (B) $\frac{Sec^3x}{3}$ (C) $\frac{tan^2x}{2}$ (D) $\frac{Sec^3x\tan x}{3}$

70. When the expression $\sqrt{a^2-x^2}$ involves in integration , we substituted:

(A) x = a cosec θ (B) x = a tan θ (C) x = a sec θ (D) x = a sin θ

71. $\int\frac{1}{x^2+9}dx=$

(A) $\frac{1}{3}\sin^{-1}\frac{x}{3}$ (B)) $\frac{1}{3}\tan^{-1}\frac{x}{3}$ (C)) $\frac{1}{3}\cos^{-1}\frac{x}{3}$ (D) $\tan^{-1}\frac{x}{3}$

72. $\int\frac{sec^2x}{\sqrt{\tan x}}$ dx is equal to:

(A) $2\sqrt{\tan x}+c$ (B) $-2\sqrt{\tan x+c}$ (C) $\sqrt{\tan x}+c$ (D) $\ell n\sqrt{\tan x}+c$

73. $\int a^{x^2}.2x$ dx is equal to:

(A) $a^{x^2}+c$ (B) $\frac{a^x+c}{2\ell na}$ (C) $a^{x^2}\ell na+c$ (D) $\frac{a^{x^2}}{\ell na}+c$

Topic IV: Integration by Parts:

74. $\int e^{ax}[af(x)+f'(x)]dx$ **(5 times)**

(A) $e^{ax}f(x)+c$ (B) $\frac{1}{a}e^{ax}f(x)+c$ (C) $\frac{1}{a}f(x)+c$ (D) $\frac{1}{a}e^{ax}+c$

75. $\int e^x(\ln x+\frac{1}{x})dx=$ **(6 times)**

(A) $\frac{e^x}{x}+c$ (B) $\ln x+e^x+c$ (C) $e^x \ln x+c$ (D) $\ln x-e^x+c$

76. $\int e^{-x}(\cos x-\sin x)dx$ **is equals:** **(4 times)**

(A) $e^{-x}\sin x+c$ (B) $e^{-x}\cos x+c$ (C) $e^x\cos x+c$ (D) $e^x\sin x+c$

77. $\int e^x\left(\frac{1}{x\sqrt{x^2-1}}+Sec^{-1}x\right)dx=$

(A) $\frac{e^x}{x\sqrt{x^2-1}}$ (B) $e^x\,Sec^{-1}x$ (C) $e^x Cosec^{-1}x$ (D) $e^x\,Cot^{-1}x$

78. $\int e^x(\cos x+\sin x)\,dx=$ **(2 times)**

(A) $e^x\cos x+c$ (B) $e^x\sin x+c$ (C) $-e^x\cos x+c$ (D) $-e^x\sin x+c$

79. $\int e^x(x+1)dx=$:

(A) e^x (B) xe^x (C) $e^x.\frac{x^2}{2}$ (D) None

80. $\int \ell nx\,dx$ **is equal to:** **(2 times)**

(A) $x\ell nx-x$ (B) $x-x\,\ell nx$ (C) $x\,\ell nx+x$ (D) $\frac{1}{x}\,\ell nx$

81. $\int e^{2x}(-\sin x+2\cos x)\,dx$ **equals:**

(A) $e^{2x}\sin$ (B) $e^{2x}\cos x$ (C) $-e^{2x}\sin x$ (D) $-e^{2x}\cos x$

82. $\int e^x\left[\frac{1}{1+x^2}+\tan^{-1}x\right]dx=$

(A) $e^x\tan x+c$ (B) $\frac{e^x}{1+x^2}+c$ (C) $e^x\sin x+c$ (D) $e^x\tan^{-1}x+c$

83. $\int e^{-x}(Cosx-Sinx)dx=$ **(2 times)**

(A) $e^x\,Cos\,x+C$ (B) $e^x\,Sin\,x+C$ (C) $e^{-x}\,Cos\,x+C$ (D) $e^{-x}\,Sinx+C$

84. $\int \ell nx\,dx=$ **(3 Times)**

(A) $x-x\ell nx+c$ (B) $x\ell nx+x+c$ (C) $\frac{1}{x}+c$ (D) $x\ell nx-x+c$

Topic VI: Area under the curve:

85. **If** $a<c<b,\int_a^b f(x)dx=$ **(4 times)**

(A) $\int_a^c f(x)dx$ (B) $\int_c^b f(x)dx$ (C) $\int_a^c f(x)dx+\int_c^b f(x)dx$ (D) $\int_a^c f(x)dx-\int_c^b f(x)dx$

86. $\int_a^x 3t^2dt$ **(4 times)**

(A) a^3-x^3 (B) a^3+x^3 (C) x^3-a^3 (D) $\frac{x^3+a^3}{3}$

87. $\int_0^1\frac{dx}{\sqrt{1-x^2}}=$ **(4 times)**

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

88. **The area bounded by cos x function from** $x=\frac{-\pi}{2}$ to $x=\frac{\pi}{2}$ **is:**

(A) 1 sq unit (B) 2 sq unit (C) 3 sq unit (D) 4 sq unit

89. $\int_0^{\frac{\pi}{4}}\sec x\tan x\,dx=$ ________ **(5 times)**

(A) $\sqrt{2}$ (B) $\sqrt{2}-1$ (C) $\sqrt{2}+1$ (D) 1

90. If $\int_{-1}^{3} x^3 dx$ is equal to: (4 times)

(A) 20 (B) 80 (C) 28 (D) 18

91. $\int_{0}^{1}(4x+k)dx = 4$, then k will be: (3 times)

(A) $-\frac{1}{3}$ (B) 0 (C) 1 (D) 2

92. $\int_{0}^{-1}\frac{1}{1+x^2}dx$ is equal to: (4 times)

(A) $\frac{\pi}{4}$ (B) $\frac{4}{\pi}$ (C) $\frac{-\pi}{4}$ (D) $\frac{-4}{\pi}$

93. $\int_{a}^{b} f(x)dx =$_______ (4 times)

(A) $-\int_{a}^{b} f(x)dx$ (B) $-\int_{b}^{a} f(x)dx$ (C) $\int_{-b}^{-a} f(x)dx$ (D) $-\int_{-a}^{-b} f(x)dx$

94. If $\int_{-2}^{1} f(x)dx = 5$, $\int_{1}^{3} f(x)dx = 3$, $\int_{-2}^{3} f(x)dx =$__________ (5 times)

(A) 2 (B) 3 (C) 5 (D) 8

95. $\int_{-\pi}^{\pi} Sinx\, dx =$ ________ (7 times)

(A) 0 (B) 2 (C) 4 (D) 6

96. $\int_{0}^{\frac{\pi}{4}} \sec^2 x\, \mathbf{dx} =$

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 0

97. $\int_{0}^{\frac{\pi}{4}} \frac{\sec^2 x}{1+\tan x} =$ (2 Times)

(a) 1 (b) 2 (c) $\ell n2$ (d) $\ell n\sqrt{2}$

98. $\int_{0}^{3}\frac{1}{\sqrt{9-x^2}}dx$ is equal to.

(a) $\frac{2}{\pi}$ (b) $-\frac{2}{\pi}$ (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

99. $\int_{0}^{1}\frac{1}{1+x^2}dx =$ (4 Times)

(a) $\frac{\pi}{4}$ (b) $\frac{2\pi}{3}$ (c) $\frac{3\pi}{4}$ (d) π

100. $\int_{0}^{\pi/2} k\cos x\, \mathbf{dx} = 4$ then k =

(a) 5 (b) 4 (c) 2 (d) 0

101. $\int_{-\infty}^{\infty}\frac{1}{1+x^2}dx =$

(a) π (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

102. $\int_0^{\pi} \cos x\, dx =$

(A) -2 (B) -1 (C) 0 (D) 2

103. $\int_0^{\frac{\pi}{2}} \sin^2 x dx =$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

104. $\int_0^{\pi/4} \cos x\, dx$ equals

(A) 1 (B) 2 (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

105. $\int_0^2 dx$ equals:

(A) 2 (B) 0 (C) 4 (D) -2

106. $\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$ is equal to :-

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

107. $\int_0^1 x^3\, dx$ is equal to

(A) 4 (B) -4 (C) $\frac{1}{4}$ (D) $\frac{-1}{4}$

108. $\int_a^b x\, dx$ equals:

(A) $\frac{b-a}{2}$ (B) $\frac{b+a}{2}$ (C) $\frac{b^2-a^2}{2}$ (D) $\frac{b^2+a^2}{2}$

109. $\int_1^4 3\sqrt{x}\, dx$ is equal to:

(A) 1 (B) 4 (C) 14 (D) 41

110. $\int_{-\pi}^{\pi} \cos x\, dx =$:

(A) 0 (B) 2 (C) -2 (D) 1

111. $\int_1^2 (x^2 + 1)\, dx$ is equal to:

(A) $\frac{3}{10}$ (B) 2 (C) $\frac{10}{3}$ (D) 0

112. $\int_0^1 \frac{1}{1+x^2}\, dx =$

(A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

113. $\int_0^{\frac{\pi}{2}} \sin^3 x \cos x\, dx =$:

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{9}$

114. $\int_a^{-x} (3t^2)\, dt =$

(A) $x^3 + a^3$ (B) $a^3 - a^3$ (C) $-x^3 - a^3$ (D) $x^2 + a^2$

115. $\int_0^1 (3-x)\,dx$ equals:

(A) $3/2$ (B) $2/3$ (C) $5/2$ (D) $2/5$

116. $\int_{-\pi/2}^{\pi/2} \cos x\,dx =$

(A) 0 (B) 1 (C) 2 (D) -2

117. $\int_0^{\pi/2} \cos x\,dx =$

(A) -2 (B) -1 (C) 0 (D) 1

Topic VII: Solution of Differential Equation:

118. The solution of differential equation $\frac{dy}{dx} = \operatorname{cosec} x \cot x$ is:

(A) y = cos ec x + c (B) y = sec x + c (C) y = - cos ec x + c (D) y = -cot x + c

119. Degree of differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 3x = 0$ is: (3 times)

(A) 1 (B) 2 (C) 0 (D) 3

120. The solution of differential equation ydx + xdy = 0 is: (4 times)

(A) ln (xy) = 0 (B) $\ln\frac{x}{y} = c$ (C) xy = c (D) $\ln\frac{y}{x} = c$

121. The solution of differential equation $\frac{dy}{dx} = -y$ is: (6 times)

(A) $y = xe^{-x}$ (B) $y = ce^{-x}$ (C) $y = e^{x}$ (D) $y = ce^{x}$

122. Solution of different equation $\frac{dy}{dx} = \cos x$ is: (3 times)

(A) y = cosx + c (B) y = tanx + c (C) y = sinx + c (D) y = cotx + c

123. The solution of differential equation $\frac{dy}{dx} = \sec^2 x$, *is*: (3 Times)

(a) y = cos x + c (b) y = sec x + c (c) $y = \cos^2 x + c$ (d) y = tan x + c

124. $y = Ce^{x^2}$ is solution of

(A) $1/x\frac{dy}{dx} - 2y = 0$ (B) $x\frac{dy}{dx} - 2y = 0$ (C) $\frac{dy}{dx} - 2y = 0$ (D) $1/x\frac{dy}{dx} - y = 0$

125. Siny cosecx $\frac{dy}{dx} = 1$

(A) cosy = cosx + c (B) y = x + c (C) cosy = cosy + c (D) –cosy = cosy + c

126. The solution of $\frac{dy}{dx} = -y$ is: (2 times)

(A) $y = e^{2x}$ (B) $y = ce^{-x}$ (C) $y = e^{x}$ (D) ce^{x}

127. Order of the differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2x = 0$ is:

(A) 0 (B) 1 (C) 2 (D) 3

128. Applying initial value conditions in solution of differential Equations, we substitute:

(A) General solution of (B) Particular solution (C) No solutio (D) Infinite solutions

129. Solution of $ydx + xdy = 0$

(A) $\frac{x}{y} = c$ (B) $\frac{y}{x} = c$ (C) $xy = c$ (D) $x + y = c$

130- $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ is equal to:

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

131- $\int e^{\tan x}\left(\sec^2 x\right) dx$ is equal to:

(A) $e^{\tan x} + c$ (B) $e^x \cdot \tan x + c$ (C) $e^x \cdot \sec x + c$ (D) $e^{\cot x + c}$

132- $\int_0^2 \left(x^2 + 1\right) dx$ is equal to:

(A) $\frac{3}{10}$ (B) $\frac{14}{3}$ (C) $\frac{5}{3}$ (D) $\frac{8}{3}$

133- $\int \frac{\log_e Tanx}{Sin2x} \cdot dx =$

(A) $\frac{1}{2}\left(\log_e\left(Tanx\right)\right)^2 + c$ (B) $\frac{1}{4}\left(\log_e\left(Tanx\right)\right)^2 + c$

(C) $\frac{1}{2}\log_e\left(Sin2x\right)^2 + c$ (D) $\frac{1}{4}\log_e\left(Sin2x\right)^2 + c$

134- $3\int_{\pi/2}^{\pi} Sinx \cdot dx =$:

(A) 1 (B) 2 (C) 3 (D) 4

135- Solution of differential equation $\left(e^x + e^{-x}\right)\frac{dy}{dx} = e^x - e^{-x}$ is y =:

(A) $\log_a(e^x - e^{-x}) + c$ (B) $\log_e(e^x + e^{-x}) + c$ (C) $\log_a(e^x - e^{-x}) + c$ (D) $\log_e(e^x - e^{-x}) + c$

136- $\int (m+1)\left[x^2 + 2x\right]^m (2x+2)dx =$:

A) $(x^2 + 2x)^{m+1} + c$ (B) $\frac{\left(x^2 + 2x\right)^{m+1}}{m+1} + c$ (C) $(x^2 + 2x)^{m-1} + c$ (D) $m\,(x^2 + 2x)^{m-1} + c$

137- $\int 3^x\, dx =$:

(A) $3^x + c$ (B) $3^x \ln 3 + c$ (C) $\frac{3^x}{\ell n 3} + c$ (D) $3\ln 3^x + c$

138- $\int_0^{\frac{\pi}{2}} \cos x\, dx =$:

(A) 0 (B) 1 (C) 2 (D) 3

139- $\int \frac{1}{f(x)} \times f'(x)\, dx =$:

(A) $\ln x + c$ (B) $\ell n\left[f'(x) + c\right]$ (C) $\frac{1}{f(x)} + c$ (D) $\ell n\left|f(x)\right| + c$

140- $\int_{-1}^{2} x\, dx =$:

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

141- $\int \sec^2 x \, dx =$:

(A) cotx + c (B) tanx + c (C) secx + c (D) cosecx + c

142- $\int \frac{\sec^2 x}{\tan x} dx - \int \frac{\operatorname{cosec}^2 x}{\cot x} dx =$:

(A) 0 (B) $2 \ell n$ tanx + c (C) $2 \ell n$cotx + c (D) ℓncotx + c

143- $\int \frac{d}{dx}\left(x^n\right)dx =$:

(A) $\frac{x^{n+1}}{n+1}+c$ (B) $nx^{n-1}+c$ (C) $\frac{x^{n+1}}{n}+c$ (D) $x^n + c$

144. $\int e^{ax}(af(x)+f'(x))\,dx =$

(a) $e^{ax}.af(x)$ (b) $e^{ax}.f'(x)$ (c) $e^{ax}.f(x)$ (d) $e^{ax}.a f'(x)$

145. $\int_0^1 \frac{1}{1+x^2} dx =$ **(2 times)**

(a) 1 (b) $\frac{\pi}{4}$ (c) 0 (d) $\frac{\pi}{2}$

146 $\int 3\sin 3x dx =$

(a) $cos3x$ (b) $-cos3x$ (c) $a\sin 3x$ (d) $9cos3x$

147. If $\int_{-1}^{5} f(x)dx = 5$, then $\int_{5}^{-1} f(x)dx =$

(a) $\frac{1}{5}$ (b) $-\frac{1}{5}$ (c) -5 (d) 5

148. $\int \frac{1}{x^2+2x+5} dx$ equals

(a) $2\tan^{-1}\left(\frac{x+1}{2}\right)+c$ (b) $2\tan^{-1}\left(\frac{x-1}{2}\right)+c$ (c) $\frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}+c\right)$ (d) $\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}+c\right)$

149. $\int \left(4-x^2\right)^{-\frac{1}{2}}(-2x)\,dx =$

(a) $2\sqrt{4-x^2}$ (b) $\frac{1}{2}\sqrt{4-x^2}$ (c) $ln\left(4-x^2\right)$ (d) $ln\sqrt{4-x^2}$

150. $\int_0^x 3t^2 dt =$:

(a) t^3 (b) $\frac{t^3}{3}$ (c) x^3 (d) 0

151. $\int_1^e ln\,x dx =$

(a) -1 (b) 0 (c) 1 (d) e

152. $\int \frac{\sin 2x}{4\sin x} dx =$

(a) $\sin 2x + c$ (b) $2\sin 2x + c$ (c) $\frac{1}{2}\sin x + c$ (d) $2\sin x + c$

153. If $\int_2^K 2\,dx = 12$, then $K = ?$:

(a) 12 (b) 16 (c) 8 (d) 4

154. Solution of Differential Equation $\frac{dy}{dx} = sec^2 x$ is:

(a) $y = cotx + c$ (b) $y = \tan x + c$ (c) $y = cosx + c$ (d) $y = -\tan x + c$

155. $\int \frac{1}{1+\cos x} dx$ equal

(a) $\cot\left(\frac{x}{2}\right) + c$ (b) $\cot\left(\frac{2}{x}\right) + c$ (c) $\tan\left(\frac{2}{x}\right) + c$ (d) $\tan\left(\frac{x}{2}\right) + c$

156. $\int_0^1 \left(5x^4 - 3x^2 + 1\right) dx$ equals

(a) 1 (b) 2 (c) 0 (d) 3

157. $\int_a^x 3t^2 . dt =$

(a) $x^3 - a^3$ (b) t^3 (c) $t^3 - a^3$ (d) 0

ANSWERS TO THE MULTIPLE CHIOCE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14
d	c	D	c	b	d	c	c	d	a	c	d	d	d
15	16	17	18	19	20	21	22	23	24	25	26	27	28
a	c	d	b	b	b	a	d	b	a	d	b	d	a
29	30	31	32	33	34	35	36	37	38	39	40	41	42
a	d	c	a	b	d	b	d	a	c	c	d	a	a
43	44	45	46	47	48	49	50	51	52	53	54	55	56
d	a	c	b	d	a	b	d	b	b	d	b	a	d
57	58	59	60	61	62	63	64	65	66	67	68	69	70
b	c	d	a	d	a	d	b	a	d	c	b	c	d
71	72	73	74	75	76	77	78	79	80	81	82	83	84
b	a	d	a	c	a	b	b	b	a	b	d	d	d
85	86	87	88	89	90	91	92	93	94	95	96	97	98
c	c	d	b	b	a	d	c	b	d	a	a	c	d
99	100	101	102	103	104	105	106	107	108	109	110	111	112
a	b	a	c	a	d	a	d	c	c	c	a	c	c
113	114	115	116	117	118	119	120	121	122	123	124	125	126
c	c	c	c	d	c	a	c	b	c	d	a	a	b
127	128	129	130	131	132	133	134	135	136	137	138	139	140
c	a	c	a	b	b	b	c	b	a	c	b	d	c
141	142	143	144	145	146	147	148	149	150	151	152	153	154
b	a	d	c	b	b	c	d	a	c	c	c	c	b
155	156	157											
d	a	a											

SHORT QUESTION'S OF CHAPTER-3 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Differential of Variables:

1. Find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ for $xy+x=4$ by differentials. (C.W) (4 times)

Sol:- $xy+x=4$

Taking differentials of both sides of the given equation, we get

$d(xy+x)=d(4)$

$d(xy)+d(x)=0$ [using d(f+g)=df+dg]

$xdy+ydx+dx=0......(i)$ [using d(f.g)= fdg+ g df]

$xdy=-(y+1)dx \Rightarrow \frac{dy}{dx}=-\frac{y+1}{x}$

$\frac{dx}{dy}=\frac{-x}{y+1}$

2. Find dy and δy of the function $y=x^2-1$ when x changes from 3 to 3.02 (H.W) (2 times)

Sol:- $y=x^2-1........(i)$ $\Rightarrow \frac{dy}{dx}=2x$

Here $x=3$ and $\delta x=dx=3.02-3=.02$

$dy=2xdx=2(3)(0.02)=0.12$

Now

$y+\delta y=(x+\delta x)^2-1$

$\delta y=(x+\delta x)^2-1-x^2+1$

$\delta y=(x+\delta x)^2-x^2$

$\delta y=(3+0.02)^2-3^2$

$\delta y=(3.02)^2-9$

$\delta y=9.12-9$

$\delta y=0.12$

3. Find δy and dy, if $y=x^2+2x$, when x changes from 2 to 1.8. (C.W) (6 times)

Sol: $y=x^2+2x.........(1)\Rightarrow \frac{dy}{dx}=2x+2$

Here x= 2 and $\delta x=dx=1.8-2=-0.2$

When x= 2, $y=(2)^2+2(2)=4+4=8$

To find δy, we have

$y+\delta y=(x+\delta x)^2+(x+\delta x)$

$8+\delta y=(2+(-0.2))^2+2\{2+(-0.2)\}$

$=(1.8)^2+2(1.8)=3.24+3.6=6.84$

$\delta y=6.84-8=-1.16$

From (1) dy=$(2x+2)dx$

$\Rightarrow dy=(2\times2+2)\times(-0.2)=6\times(-0.2)=-1.2$

4. **Using differentials to find $\frac{dy}{dx}$ if $x^2+2y^2=16$.** **(H.W) (4 times)**

Sol:- Taking differentials of both sides of the given equation, we get

$d(x^2+2y^2)=d(16)$

$d(x^2)+d(2y^2)=0$

$2xdx+2(2ydy)=0$............(1)

$2ydy=-xdx \Rightarrow dy=-\frac{x}{2y}dx \Rightarrow \frac{dy}{dx}=-\frac{x}{2y}$

From (1) $xdx=-2ydy \Rightarrow dx=-\frac{2y}{x}dy \Rightarrow \frac{dx}{dy}=-\frac{2y}{x}$

5. **Using differentional to find the value of $\sqrt[4]{17}$.** **(H.W) (2 times)**

Sol Let $y=\sqrt[4]{x}$

put $x=16$ and $dx=\delta x=1$

then

$y=\sqrt[4]{16}=(16)^{1/4}=2$

Now

$dy=dx^{1/4}$

$dy=\frac{1}{4}x^{1/4-1}$

$dy=\frac{1}{4}x^{-3/4}$

$dy=\frac{1}{4x^{3/4}}$

$dy=\frac{1}{4(2^4)^{3/4}}$

$dy=\frac{1}{4(8)}=\frac{1}{32}=0.0312$

Now

$y+\delta y=\sqrt[4]{x+\delta n}$

or

$\sqrt[4]{x+\delta n}=y+\delta y$

$\sqrt[4]{x+dx}\approx y+dy$

$\sqrt[4]{18+1}\approx 2+0.0312$

$\sqrt[4]{17}\approx 2.0312$ Which required.

6. **Use differential to calculate Cos29⁰.** **(C.W)**

Sol Cos29^0

$f(x+\delta x)=Cos29^0 = Cos(30^0-1^0)$

Let y = cosx Let x = 30^0

$dx = \delta x = -1^0 = -0.01745$

Taking differential on both sides.

$dy = d\cos x$
$dy = -\sin x dx$
$dy = -\sin 30^0(-0.01745)$
$dy = 0.5\,(0.01745)$
$dy = 0.0087$

Now $f(x+\delta x) \approx y + dy$
$f(x+\delta x) \approx \cos x + dy$
$\cos 29^0 \approx \cos 30^0 + 0.0087$
$\cos 29^0 \approx 0.866 + 0.0087$
$\cos 29^0 \approx 0.8747$

7. Find δy if when $y = \sqrt{x}$ x changes from 4 to 4.41. **(H.W) (5 times)**

Sol Given

$y = \sqrt{x}$
$y + \delta x = \sqrt{x+\delta x}$
$\delta y = \sqrt{x+\delta x} - y$
$\delta y = \sqrt{x+\delta x} - \sqrt{x}$
$\delta y = \sqrt{4.41} - \sqrt{4}$
$\delta y = 2.1 - 2$
$\delta y = 0.1$

x changes from 4 to 4.41
x = 4
$\delta x = 4.41 - 4$
$\delta x = 0.41$

Topic II: Anti-Derivative (Integration):

8. Evaluate $\int \frac{1-x^2}{1+x^2}dx$ **(H.W) (3 times)**

Sol:- $\int \frac{1-x^2}{1+x^2}dx = \int\left[\frac{2-x^2-1}{1+x^2}\right]dx = \int\left[\frac{2-(x^2+1)}{1+x^2}\right]dx$

$= \int\frac{2}{1+x^2}dx - \int\frac{x^2+1}{x^2+1}dx = 2\int\frac{1}{x^2+1}dx - \int 1dx \quad = 2\tan^{-1}x - x + c$

9. Evaluate $\int \frac{3x+2}{\sqrt{x}}dx$ **(H.W) (4 times)**

Sol:- $\int\frac{3x+2}{\sqrt{x}}dx = \int\left[\frac{3x}{\sqrt{x}} + \frac{2}{\sqrt{x}}\right]dx = 3\int\sqrt{x}dx + 2\int\frac{1}{\sqrt{x}}dx$

$= 3\int x^{\frac{1}{2}}dx + 2\int x^{\frac{-1}{2}}dx = 3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 2\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$

$= 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = 2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + c$

10. Evaluate $\int \sin^2 x dx$ **(H.W) (2 times)**

Sol:- $\int \sin^2 x dx \quad = \int\left(\frac{1-\cos 2x}{2}\right)dx$

$= \frac{1}{2}\int 1dx - \frac{1}{2}\int \cos 2x dx$

$=\frac{1}{2}x-\frac{1}{2}\frac{\sin 2x}{2}+c=\frac{1}{2}x-\frac{1}{4}\sin 2x+c$

11. Evaluate $\int \tan^2 x\, dx$ **(C.W) (10 times)**

Sol: $\int \tan^2 x dx$

$=\int (\sec^2 x-1)\, dx$ $\quad \because \tan^2 x=\sec^2 x-1$

$=\tan x - x + c$ $\quad \because \int \sec^2 x\, dx=\tan x$

12. Evaluate $\int (\sqrt{x}-\frac{1}{\sqrt{x}})^2\, dx$ **(H.W) (3 times)**

Sol: Given: $\int (\sqrt{x}-\frac{1}{\sqrt{x}})^2\, dx$

$=\int (\sqrt{x})^2+\left(\frac{2}{\sqrt{x}}\right)^2-2\sqrt{x}.\frac{1}{\sqrt{x}}\, dx$

$=\int \left(x+\frac{1}{x}-2\right)dx$ $\quad \because \int \frac{1}{x}dx=\ell nx$

$=\int x dx+\int \frac{1}{x}\, dx-\int 2dx$

$=\frac{x^2}{2}+\ell nx-2x+c$

13. Evaluate $\int \text{Sec}^4 x\, dx.$ **(H.W) (3 times)**

Sol: $\int \text{Sec}^4 x\, dx$

$=\int (\sec^2 x)(\sec^2 x)\, dx$

$=\int \sec^2 x(1+\tan^2 x)dx$

$=\int (\sec^2 x+\tan^2 x.\sec^2 x)\, dx$

$=\int \sec^2 x\, dx+\int \tan^2 x.\sec^2 x dx$

$=\tan x+\frac{\tan^3 x}{3}+c.$

14. Evaluate $\int Cos 3x\ Sin 2x\, dx$ **(H.W) (4 times)**

Sol Given $\int Cos 3x\ Sin 2x\, dx$

$=\frac{1}{2}\int 2Cos 3x\ Sin 2x\, dx$

$\because 2Cos\alpha Sin\beta=Sin(\alpha+\beta)-Sin(\alpha-\beta)$

$=\frac{1}{2}\int (Sin(3x+2x)-Sin(3x-2x))dx$

$=\frac{1}{2}\int Sin 5x\, dx-\frac{1}{2}\int Sin x\, dx$ $\quad \because \int Sin x\, dx=-Cosx+c$

$=\frac{1}{2}\left(\frac{-Cos 5x}{5}\right)-\frac{1}{2}(-Cosx)+c$

$=-\frac{Cos5x}{10}+\frac{Cosx}{2}+c$

15. Evaluate $\int (2x+3)^{1/2}\, dx$ **(H.W)**

Sol

Given

$\int (2x+3)^{1/2}\, dx$

$=\frac{1}{2}\int (2x+3)^{1/2}\, 2dx$

$=\frac{1}{2}\ \frac{(2x+3)^{1/2+1}}{1/2+1}+c$

Aside

Let $f(x)=2x+3$

$f'(x)=2+0$

$f'(x)=2$

$$\because \int f(x)^n f'(x)dx = \frac{f(x)^{n+1}}{n+1} + c$$

$$= \frac{1}{2} \frac{(2x+3)^{3/2}}{3/2} + c$$

$$= \frac{1}{3} (2x+3)^{3/2} + c$$

16. **Evaluate** $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$; $(x > 0)$ **(C.W) (2 times)**

Sol **Given**

$$\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$$

$$= \int \frac{1+(\sqrt{x})^2 - 2\sqrt{x}}{\sqrt{x}} dx$$

$$= \int \frac{1+x-2\sqrt{x}}{\sqrt{x}} dx$$

$$= \int \left(\frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}} - \frac{2\sqrt{x}}{\sqrt{x}}\right) dx$$

$$= \int \left(x^{-1/2} + x^{1/2} - 2\right) dx$$

$$= \int x^{-1/2} dx + \int x^{1/2} dx - 2\int 1 dx$$

$$= \frac{x^{1/2}}{1/2} + \frac{x^{3/2}}{3/2} - 2x + c$$

$$= 2\sqrt{x} + \frac{2}{3}x^{2/3} - 2x + c \text{ Ans.}$$

Topic III: Substitution Method:

17. **Evaluate** $\int \frac{e^x}{e^x+3} dx$. **(H.W) (3 times)**

Sol:- $= \int \frac{e^x}{e^x+3} dx$ put $u = e^x + 3 \Rightarrow du = e^x dx$

$$= \int \frac{1}{u} du = \ln u + c = \ln(e^x+3) + c$$

18. **Evaluate** $\int \operatorname{cosec} x dx$ **Multiplying and dividing by** $(\operatorname{cosec}x - \cot x)$ **(C.W)**

Sol:- $\int \operatorname{cosec} x dx = \int \frac{\operatorname{cosec}x(\operatorname{cosec}x - \cot x)}{(\operatorname{cosec}x - \cot x)} dx$

Put cosec x - cot x= t, then (-cosec x cot x + cosec^2 x) dx= dt

Or cosec x(cosec x – cot x) dx= dt

So $\int \frac{\operatorname{cosec}x(\operatorname{cosec}x - \cot x)}{(\operatorname{cosec}x - \cot x)} dx = \int \frac{1}{t} dt = \ln|t| + c$

Thus $\int \operatorname{cosec}x dx = \ln|\operatorname{cosec}x - \cot x| + c$ $[\because t = \operatorname{cosexc} - \cot x]$

19. **Evaluate :** $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx.$ **(C.W) (3 times)**

Sol:- $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx = \int (\tan x)^{-\frac{1}{2}} \left(\sec^2 x\right) dx$

$$= \frac{(\tan)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c = \frac{(\tan)^{\frac{1}{2}}}{\frac{1}{2}} + c = 2\sqrt{\tan x} + c$$

20. **Evaluate** $\int \frac{ax+b}{ax^2+2bx+c} dx$ **Multiplying and dividing by 2 (H.W) (4 times)**

Sol:- $\int \frac{ax+b}{ax^2+2bx+c} dx = \frac{1}{2} \int \frac{2ax+2b}{ax^2+2bx+c} dx$

$= \frac{1}{2} \ln\left|ax^2+2bx+c\right| + c_1$ $\left[\because \frac{d}{dx}\left(ax^2+2bx+c\right) = 2ax+2b\right]$

21. **Evaluate** $\int \frac{dx}{x^2+4x+13}.$ **(C.W)**

Sol:- $\int \frac{dx}{x^2+4x+13}$

$= \int \frac{dx}{x^2+4x+4+9}$ $\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$= \int \frac{dx}{(x+2)^2+(3)^2}$

$= \frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right) + c$

22. **Evaluate** $\int \frac{1}{x \ln x} dx.$ **(H.W) (5 times)**

Sol:- $\int \frac{1}{x \ln x} dx$

$= \int \frac{\frac{1}{x}}{\ln x} dx$ (dividing top and bottom by x)

$= \ln(\ln x) + c$ $\left[\because \int \frac{f'(x)}{f(x)} dx = \ln f(x)\right]$

23. **Evaluate** $\int \frac{x^2}{4+x^2}$ **dx. (H.W) (10 times)**

Sol: $\int \frac{x^2}{4+x^2} dx$

$= \int \frac{(4+x^2)-4}{4+x^2} dx$

$= \int \left(1 - \frac{4}{4+x^2}\right) dx = \int 1\, dx - 4 \frac{1}{4+x^2} dx$

$= x - \frac{4}{2}\tan^{-1}\frac{x}{2} + c$ $\because \int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$= x - 2\tan^{-1}\frac{x}{2} + c$

24. **Evaluate** $\int x\sqrt{x^2-1}$ **dx (C.W) (5 times)**

Sol: Given $\int x\sqrt{x^2-1}\, dx$

$= \int \sqrt{x^2-1}\,.xdx$

$= \frac{1}{2}\int (x^2-1)^{1/2}\, 2x\, dx$

$= \frac{1}{2}\frac{(x^2-1)^{3/2}}{3/2} + c$

$= \frac{2}{2\,(3)}(x^2-1)^{3/2} + c = \frac{1}{3}(x^2-1)^{3/2} + c$

25. **Evaluate** $\int \frac{x}{\sqrt{4+x^2}}dx$ **(C.W) (3 times)**

Sol **Given**

$\int \frac{x}{\sqrt{4+x^2}}dx$

$= \int (4+x^2)^{-1/2}\, x\,dx$

$= \frac{1}{2}\int (4+x^2)^{-1/2}\, 2x\,dx$

Aside

$\because Let \quad f(x) = 4+x^2$

$f'(x) = 0+2x$

$f'(x) = 2x$

$\because \int f(x)^n f'(x)dx = \frac{f(x)^{n+1}}{n+1} + c$

$= \frac{1}{2}\frac{(4+x^2)^{-1/2+1}}{\frac{-1}{2}+1} + c$

$= \frac{1}{2}\frac{(4+x^2)^{1/2}}{1/2} + c \quad , \quad = \frac{2}{2}\sqrt{4+x^2} + c = \sqrt{4+x^2} + c$

26. **Evaluate** $\int \frac{Cot\sqrt{x}}{\sqrt{x}}\,dx \;,\; x > 0$ **(C.W) (4 times)**

Sol **Given**

$\int \frac{Cot\sqrt{x}}{\sqrt{x}}\,dx$

$= \int Cot\sqrt{x}\left(\frac{1}{\sqrt{x}}\right)dx$ **Let** $t = \sqrt{x} \Rightarrow t^2 = x$

$= \int \cot\frac{2tdt}{t}$ $\Rightarrow 2tdt = dx$

$= 2\int \cot dt$ $\because \int Cot\,x\,dx = \ln Sinx + c$

$= 2\,\ln(S\text{int}) + c$

$= 2\,\ln(Sin\sqrt{x}) + c$ **Ans.**

27. **Evaluate** $\int \frac{Sin\theta}{1+Cos^2\theta}\,d\theta$ **(H.W) (3 times)**

Sol **Given**

$\int \frac{Sin\theta}{1+Cos^2\theta}\,d\theta$

$= \int \frac{Sin\theta}{1+(Cos\theta)^2}\,d\theta$ **Let** $t = Cos\theta$

$= \int \frac{-dt}{1+t^2}$ $\Rightarrow dt = -\sin\theta d\theta$

$= -\int \frac{dt}{1+t^2}$ $\qquad \because \int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c$

$= -\tan^{-1}(t) + c \qquad = -\tan^{-1}(Cos\theta) + c$ Ans.

Topic IV: Integration by Parts:

28. Evaluate $\int \sin^{-1} x dx$ **(H.W) (3 times)**

Sol:- $\int \sin^{-1} x dx$

$= \int (\sin^{-1}x)(1)\,dx$

$= (\sin^{-1}x)(x) - \int x.\frac{1}{\sqrt{1-x^2}}\,dx$

$= x\sin^{-1}x + \frac{1}{2}\int (1-x^2)^{\frac{1}{2}}(-2x)\,dx$

$= x\sin^{-1}x + \frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c = x\sin^{-1}x + \sqrt{1-x^2} + c$

29. Evaluate $\int \ln x dx$ **(C.W)**

Sol:- Let $I = \int \ln x dx = \int 1.\ln x dx$

Integrating by parts, we have

$= \ln x.(x) - \int (x)\left(\frac{1}{x}\right)dx$

$= x\ln x - \int 1 dx = x\ln x - x + c$

30. Evaluate $\int e^{2x}$ **(-sin x + 2 cos x) dx** **(H.W) (3 times)**

Sol: Given $\int e^{2x}$ **(-sin x + 2 cos x) dx**

or $\int e^{2x}[2\,cosx + (-sinx)]$ dx

$\because \int e^{ax}[af(x) + f'(x)]\,dx = e^{ax}f(x) + c$

$= e^{2x} \cos x + c$

31. Evaluate $\int e^x\left(\frac{1}{x} + \ell nx\right)$ **dx** **(H.W) (6 times)**

Sol: Let: $\int e^x\left(\frac{1}{x} + \ell nx\right)$ dx

or $= \int e^x\left[\ell nx + \frac{1}{x}\right] dx$ $\qquad \because \int e^x[f(x) + f'(X)]\,dx = e^x f(X) + c$

$= e^x\,\ell nx + c$

32. Evaluate $\int x\ln x dx$. **(H.W) (8 times)**

Sol:- Let $I = \int x\ln x dx$

Integrating by parts, we have

$= \ln x.\left(\frac{x^2}{2}\right) - \int\left(\frac{x^2}{2}\right)\times\left(\frac{1}{x}\right)dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\left(\frac{x^2}{2}\right) + c = \frac{1}{2}x^2\left(\ln x - \frac{1}{2}\right) + c$

33. **Evaluate** $\int e^{2x}\left(\frac{3Sinx-Cosx}{Sin^2x}\right)dx$ **(H.W) (2 times)**

Sol Given

$$\int e^{2x}\left(\frac{3Sinx-Cosx}{Sin^2x}\right)dx$$

$$=\int e^{3x}\left[3\frac{Sinx}{Sin^2x}-\frac{Cosx}{Sin^2x}\right]dx$$

$$=\int e^{3x}\left[\frac{3}{Sinx}-\frac{Cosx}{Sinx\,Sinx}\right]dx$$

$$=\int e^{3x}\left[3Co\sec x-Co\sec xCotx\right]dx$$

$$\because f(x)=Co\sec x$$

$$f'(x)=-Co\sec x\,Cotx$$

$e^{3x}Co\sec x+c$ $\qquad \because \int e^{ax}\left[af(x)+f'(x)\right]dx=e^{ax}f(x)+c$

34. **Evaluate** $\int x\,Cosx\,dx$ **(C.W)**

Sol **Given**

$$\int \underset{I}{x}\,\underset{II}{Cosx}\,dx$$

Integration by parts

$$= x\,Sin\,x - \int 1.\,Sin\,x\,dx$$

$$= x\,Sin\,x - \int Sin\,x\,dx$$

$$= xSinx - (-Cosx) + c$$

$$= xSinx + Cosx + c \quad \text{Ans.}$$

Topic V: Integration by Partial Fraction:

35. **Evaluate** $\int\frac{3x+1}{x^2-x-6}dx$ **(C.W) (2 times)**

Sol;- $\int\frac{3x+1}{x^2-x-6}dx$ ....(i)

Since $x^2-x-6=x^2-3x+2x-6=x(x-3)+2(x-3)=(x+2)(x-3)$

Let $\frac{3x+1}{(x+2)(x-3)}=\frac{A}{x+2}+\frac{B}{x-3}$

Multiply both sides by L.C.M=(x+2)(x-3), we have

$3x+1=A(x-3)+B(x+2)$..........(1)

Put x=3 in (1)

$3(3)+1=A(3-3)+B(3+2)$

$9+1=A(0)+B(5)\Rightarrow 10=5B\Rightarrow B=\frac{10}{5}\Rightarrow B=2$

Put x= -1 in (1)

$3(-2)+1=A(-2-3)+B(-2+2)$

$-6+1=A(-5)+B(0)\Rightarrow -5=-5A+0\Rightarrow A=1$

Hence (1) can be written as

$$\int\frac{3x+1}{x^2-x-6}dx=\int\left(\frac{1}{x+2}+\frac{2}{x-3}\right)dx$$

$$=\int\frac{1}{x+2}dx+2\int\frac{1}{x-3}dx$$

$$=\ln|x+2|+2\ln|x-3|+c$$

36. Find $\int\frac{(a-b)x}{(x-a)(x-b)}dx$ **(H.W) (2 times)**

Sol: First by partial fraction

$$\frac{(a-b)x}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b} \quad \text{.......... (i)}$$

(a – b)x = A (x – b) + B (x – a) .... (ii)

Put x – a = 0 in .... (ii)

$\cancel{(a-b)}\, a = A\, \cancel{(a-b)} \Rightarrow A = a$

Put x – b = 0 in .... (ii)

(a – b) b = B (b – a)

$$\frac{-\cancel{(b-a)}b}{\cancel{(b-a)}}=B \Rightarrow B=-b$$

Put the value of A and B in ..(i)

$$\frac{(a-b)x}{(x-a)(a-b)}=\frac{a}{x-a}+\frac{(-b)}{x-b}$$

By integrating

$$\int\frac{(a-b)x}{(x-a)(x-b)}=a\int\frac{1}{x-a}dx-b\int\frac{1}{x-b}dx$$

$$=a\ln|x-a|-b\ln|x-b|+c$$

Topic VI: Area under the curve:

37. Evaluate : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos t\, dt$ **(H.W) (6 times)**

Ans:- $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos t\, dt$

$$=[\sin t]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$=\left\{\sin\left(\frac{\pi}{3}\right)-\sin\left(\frac{\pi}{6}\right)\right\}$$

$$=\frac{\sqrt{3}}{2}-\frac{1}{2}=\frac{\sqrt{3}-1}{2}$$

38. **Find the area between x-axis and curve $y=x^2+1$ from $x=+1$ to $x=2$**

(H.W) (2 times)

Sol:- The required area = $\int_1^2 (x^2+1)dx=\int_1^2 x^2dx+\int_1^2 1dx$

$$=\left|\frac{x^3}{3}\right|_1^2+|x|_1^2=\left[\frac{2^3}{3}-\frac{1^3}{3}\right]+(2-1)=\left[\frac{8}{3}-\frac{1}{3}\right]+1=\frac{7}{3}+1=\frac{10}{3} \text{ square units}$$

39. **Evaluate :** $\int_0^3 \frac{dy}{x^2+9}dx$ **(H.W) (5 times)**

Sol:- $\int_0^3 \frac{dy}{x^2+9}dx$

$=\int_0^3 \frac{1}{x^2+(3)^2}dx$

$=\frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^3$

$=\frac{1}{3}\left[\tan^{-1}\frac{3}{3}-\tan^{-1}\frac{0}{3}\right]=\frac{1}{3}\left[\tan^{-1}1-0\right]=\frac{1}{3}\left[\frac{\pi}{4}\right]=\frac{\pi}{12}$

40. **Find the area below the curve $y=3\sqrt{x}$ and above the x-axis between x=1 and x=4.** **(H.W) (2 times)**

Sol:- The required area $=\int_1^4 3\sqrt{x}dx=3\int_1^4 x^{\frac{1}{2}}dx$

$=3\left|\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right|_1^4=3\left|\frac{2}{3}x^{\frac{3}{2}}\right|_1^4$

$=2\left|x^{\frac{3}{2}}\right|_1^4=2\left[4^{\frac{3}{2}}-1^{\frac{3}{2}}\right]$

$=2\left[4^{\frac{3}{2}}-1^{\frac{3}{2}}\right]$

$=2|8-1|=14$ square units.

41. **Evaluate :** $\int_0^{\frac{\pi}{6}} \cos^3\theta d\theta.$ **(H.W)**

Sol:- $\int_0^{\frac{\pi}{6}} \cos\theta\cos^2\theta d\theta.$

$=\int_0^{\frac{\pi}{6}} \cos\theta\left(1-\sin^2\theta\right)d\theta$

$=\int_0^{\frac{\pi}{6}} \cos\theta d\theta-\int_0^{\frac{\pi}{6}} \cos\theta\sin^2\theta d\theta$

$=\left|Sin\theta\right|_0^{\pi/6}-\left|\frac{Sin^3\theta}{3}\right|_0^{\pi/6}$

$=\left\{Sin\left(\frac{\pi}{6}\right)-Sin(0)\right\}-\frac{1}{3}\left\{Sin^3\frac{\pi}{6}-0\right\}$

$$=\left(\frac{1}{2}-0\right)-\frac{1}{3}\left(\left(\frac{1}{2}\right)^3-0\right)$$

$$=\frac{1}{2}-\frac{1}{3}\left(\frac{1}{8}\right)=\frac{1}{2}-\frac{1}{24}=\frac{12-1}{24}=\frac{11}{24}$$

42. **Find the area above the x-axis and under the curve $y=5-x^2$ from $x=-1$ to $x=2$.** **(C.W) (3 times)**

Sol:- The required area $=\int_{-1}^{2}(5-x^2)dx=|5x|_{-1}^{2}-\left|\frac{x^3}{3}\right|_{-1}^{2}$

$$=[5(2)-(-1)]-\left[\frac{2^3}{3}-\frac{(-1)^3}{3}\right]$$

$$=(10+1)-\left(\frac{8}{3}+\frac{1}{3}\right)$$

$$=11-\frac{9}{3}$$

$=11-3=8$ square units.

43. **Evaluate** $\int_1^2\frac{x}{x^2+2}dx$. **(H.W) (4 times)**

Sol:- $\int_1^2\frac{x}{x^2+2}dx$

$$=\frac{1}{2}\int_1^2\frac{2x}{x^2+2}dx$$

$$=\frac{1}{2}\left[\ln(x^2+2)\right]_1^2$$

$$=\frac{1}{2}\left[\ln(2^2+2)-\ln(1^2+2)\right]$$

$$=\frac{1}{2}[\ln 6-\ln 3]=\frac{1}{2}\ln\left(\frac{6}{3}\right)$$

$$=\frac{1}{2}\ln 2$$

44. **Evaluate : $\int_0^{\pi/4}$ sec x (sec x + tan x).** **(C.W) (7 times)**

Sol: Given $\int_0^{\pi/4}$ sec x (sec x + tan x) dx

$$=\int_0^{\pi/4}\sec^2 x\,dx+\int_0^{\pi/4}\sec x\tan x\,dx$$

$$=|\tan x|_0^{\pi/4}+|\sec x|_0^{\pi/4}$$

$$=\left(\tan\frac{\pi}{4}-\tan 0\right)+\left(\sec\frac{\pi}{4}-\sec 0\right)$$

$$=(1-0)+(\sqrt{2}-1)$$

$= 1 + \sqrt{2} - 1 = \sqrt{2}$

45. Find the area between x – axis and the curve $y = 4x - x^2$. (C.W) (5 times)

Sol: Given $y = 4x - x^2$ (1)

Put y = 0 in eq. (1)

$4x - x^2 = 0$

$x(4 - x) = 0$

$x = 0,\ 4 - x = 0$

$x = 0,\ x = 4$ Then a = 0 , b = 4

now Area $= \int_a^b f(X)\,dx.$

$= \int_0^4 (4x - x^2)\,dx$

$= \left|4.\frac{x^2}{2} - \frac{x^3}{3}\right|_0^4 = \left|2x^2 - \frac{x^3}{3}\right|_0^4$

$= \left[2(4)^2 - \frac{(4)^3}{3}\right] - \left[2(0^2) - \frac{(0)^3}{3}\right]$

$= 32 - \frac{64}{3} - 0 = \frac{96-64}{3} = \frac{32}{3}$ square units

46. Evaluate: $\int_{-2}^{0} \frac{1}{(2x-1)^2}\,dx.$ (H.W) (4 times)

Sol: $\int_{-2}^{0} \frac{1}{(2x-1)^2}\,dx$

$= \frac{1}{2}\int_{-2}^{0} \frac{1}{(2x-1)^2}\,2dx$

$= \frac{1}{2}\left|\frac{(2x-1)^{-1}}{-1}\right|_{-2}^{0} = -\frac{1}{2}\left|\frac{1}{2x-1}\right|_{-2}^{0}$

$= -\frac{1}{2}\left[\frac{1}{2(0)-1} - \frac{1}{2(-2)-1}\right] = -\frac{1}{2}\left[-1 - \frac{1}{-5}\right]$

$= -\frac{1}{2}\left[-1 + \frac{1}{5}\right] = \frac{-1}{2}\left[\frac{-4}{5}\right] = \frac{2}{5}$

47. Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx$ (C.W) (4 times)

Sol: $= \int_0^{\frac{\pi}{4}} \frac{1}{1+Sinx} \times \frac{1-sinx}{1-sinx}\,dx$

$= \int_0^{\frac{\pi}{4}} \frac{1-sinx}{1-sin^2x}\,dx = \int_0^{\frac{\pi}{4}} \frac{1-sinx}{cos^2x}\,dx$

$= \int_0^{\frac{\pi}{4}} \left[\frac{1}{cos^2x} - \frac{Sin\,x}{cCos^2x}\right]dx = \int_0^{\frac{\pi}{4}} (\sec^2x - \sec x\tan x)\,dx$

$= \int_0^{\frac{\pi}{4}} \sec^2 x\,dx - \int_0^{\frac{\pi}{4}} \sec x\tan x\,dx$

$= |\tan x|_0^{\pi/4} - |\sec x|_0^{\pi/4}$

$= (\tan\frac{\pi}{4} - \tan 0) - [\sec\frac{\pi}{4} - \sec 0]$

$= (1-0) - (\sqrt{2} - 1) = 1 - \sqrt{2} + 1 = 2 - \sqrt{2}$

48. **Compute :** $\int_{-6}^{2} \sqrt{3-x}\, dx.$ **(H.W) (4 times)**

Sol: Let: $\int_{-6}^{2} \sqrt{3-x}\, dx.$

$= -\int_{-6}^{2} (3-x)^{1/2}(-1)\, dx$

$= -\left|\frac{(3-x)^{3/2}}{3/2}\right| = -\frac{2}{3}[(3-x)^{3/2}]_{-6}^{2}$

$= -\frac{2}{3}\left[(3-2)^{3/2} - (3-(-6)^{3/2}\right] = -\frac{2}{3}\left[(1)^{3/2} - (3+6)^{3/2}\right]$

$= -\frac{2}{3}\left[1 - (3^2)^{3/2}\right] = -\frac{2}{3}[1 - (3)^3] = -\frac{2}{3}[1-27]$

$= -\frac{2}{3}(-26) = \frac{52}{3}$

49. **Find** $\int_1^2 (x^2+1)\, dx$ **(H.W)** **(2 times)**

Sol **Given**

$\int_1^2 (x^2+1)\, dx$ $\qquad \because \int x^n\, dx = \frac{x^{n+1}}{n+1} + c$

$= \left|\frac{x^3}{3} + x\right|_1^2$

$= \left(\frac{(2)^3}{3} + 2\right) - \left(\frac{(1)^3}{3} + 1\right) = \left(\frac{8}{3} + 2\right) - \left(\frac{1}{3} + 1\right)$

$= \left(\frac{8+6}{3}\right) - \left(\frac{1+3}{3}\right)$

$= \frac{14}{3} - \frac{4}{3} = \frac{14-4}{3} = \frac{10}{3}$

50. **Evaluate** $\int_0^{\pi/6} \underset{I}{x}\ \underset{II}{Cosx}\ dx$ **(C.W)**

Sol **Given**

$\int_0^{\pi/6} \underset{I}{x}\ \underset{II}{Cosx}\ dx$

Taking integration by parts.

$= |x\,Sinx|_0^{\pi/6} - \int_0^{\pi/6} 1 \,.\, Sinx\, dx$

$= \left[\frac{\pi}{6}\cdot\left(Sin\frac{\pi}{6}\right) - 0\right] - |-Cosx|_0^{\pi/6}$

$$= \frac{\pi}{6}\left(\frac{1}{2}\right) + \left|Cosx\right|_0^{\pi/6}$$
$$= \frac{\pi}{12} + \left(Cos\frac{\pi}{6} - Cos\,0\right)$$
$$= \frac{\pi}{12} + \left(\frac{\sqrt{3}}{2} - 1\right)$$
$$= \frac{\pi + 6\sqrt{3} - 12}{12}$$

Topic VII: Solution of Differential Equation:

51. **Solve differential equation** $\frac{dy}{dx} = -y$. **(H.W) (2 times)**

Sol:- $\frac{dy}{dx} = -y$

Separating variables, we have

$\frac{1}{y}dy = -dx$

Integrating both sides, we have

$\int\frac{1}{y}dy = -\int 1dx$

$\Rightarrow \ln y = -x + c_1$

$\Rightarrow y = e^{-x+c_1}$

$y = e^{-x}e^{c_1} = ce^{-x} \quad \therefore e^{c_1} = c$

52. **Solve the differential equation** $ydx + xdy = 0$ **(C.W) (4 times)**

Sol:- $ydx + xdy = 0$

Separating variables, we have

$\Rightarrow ydx = -xdy$

$\frac{1}{x}dx = -\frac{1}{y}dy$

Integrating both sides, we have

$\int\frac{1}{x}dx = -\int\frac{1}{y}dy$

$\ln x = -\ln y + \ln c \Rightarrow \ln x + \ln y = \ln c$

$y = \ln c$

$\ln xy = \ln c \Rightarrow xy = c$

53. **Solve the differential equation** $\frac{dy}{dx} = \frac{y}{x^2}$ **(H.W)**

Sol:- $\frac{dy}{dx} = \frac{y}{x^2}$

Separating variables, we have

$\int\frac{1}{y}dy = \int\frac{1}{x^2}dx \Rightarrow \ln y = \int x^{-2}(1)dx$

$\ln y = \frac{x^{-2+1}}{-2+1} + c_1 \Rightarrow \ln y = -\frac{1}{x} + c_1$

$y = e^{-\frac{1}{x}+c_1} = e^{-\frac{1}{x}}e^{c_1} \Rightarrow y = ce^{-\frac{1}{x}}$

54. Solve the differential equation: $\left(e^x+e^{-x}\right)\frac{dy}{dx}=e^x+e^{-x}$. **(H.W)**

Sol:- $\left(e^x+e^{-x}\right)\frac{dy}{dx}=e^x+e^{-x}$

Separating variables, we have

$$dy=\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)dx$$

Integrating both sides, we have

$$\int dy=\int\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)dx$$

$$y=\ln\left(e^x+e^{-x}\right)+c \qquad \left[\because \frac{d}{dx}\left(e^x+e^{-x}\right)=e^x-e^{-x}\right]$$

55. Solve the differential equation: $xdy+y(x-1)dx=0$. **(H.W) (2 times)**

Sol:- $xdy+y(x-1)dx=0$

$$xdy=-y(x-1)dx$$

Separating variables, we have

$$\frac{1}{y}dy=-\left(\frac{x-1}{x}\right)dx \Rightarrow \frac{1}{y}dy=-\left(1-\frac{1}{x}\right)dx$$

Integrating on both sides, we have

$$\int\frac{1}{y}dy=-\int\left(1-\frac{1}{x}\right)dx \Rightarrow \int\frac{1}{y}dy$$

$$=-\int 1dx+\int\frac{1}{x}dx$$

$$\ln y=-x+\ln x+\ln c$$

$$\ln y=-x+\ln cx$$

$$\ln y-\ln cx=-x$$

$$\ln\left(\frac{y}{cx}\right)=-x \Rightarrow \frac{y}{cx}=e^{-x} \Rightarrow y=cxe^{-x}$$

$$\ln\left(\frac{y}{cx}\right)=-x \Rightarrow \frac{x}{cx}=e^{-x} \Rightarrow y=cxe^{-x}$$

56. Solve the differential equation $\frac{1}{x}\frac{dy}{dx}=\frac{\left(1+y^2\right)}{2}$ **(C.W)**

Sol **Given**

$$\frac{1}{x}\frac{dy}{dx}=\frac{1+y^2}{2}$$

Separating Variables

$$\frac{1}{1+y^2}\,dy=\frac{1}{2}\,x\,dx$$

Taking integral on both sides

$$\int\frac{1}{1+y^2}\,dy=\frac{1}{2}\cdot\int x\,dx$$

$$\tan^{-1}y=\frac{1}{2}\cdot\left(\frac{x^2}{2}\right)+c$$

$$\tan^{-1} y = \frac{x^2}{4} + c$$

$$y = \tan\left(\frac{x^2}{4} + c\right) \text{ Ans.}$$

57. **Evaluate** $\int \frac{xe^x}{(1+x)^2} dx$ **(C.W)**

Sol: $\int \frac{xe^x}{(1+x)^2} dx = \int e^x\left(\frac{1+x+1}{(1+x)^2}\right) dx = \int e^x\left(\frac{1+x}{(1+x)^2} - \frac{1}{(1+x)^2}\right) dx$

$$= \int e^x\left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right) dx$$

Using formula $\int e^x[f(x) + f'(x)]dx = e^x f(x) + c$

$$= e^x\left(\frac{1}{1+x}\right) + c$$

$$= \frac{e^x}{1+x} + c \qquad \text{Ans.}$$

58. **Find the area bounded by the curve $y = x^3 + 3x^2$ and x – axis.** **(H.W)**

Sol: Given $y = x^3 + 3x^2$

Putting $y = 0$, we have

$x^3 + 3x^2 = 0$

$x^2(x + 3) = 0$

$x^2 + 3x^2 = 0$

$x^2(x + 3) = 0$

$x^2 = 0 \quad , \quad x + 3 = 0$

$x = 0 \quad , \quad x = -3$

The curve cuts the x – axis at point(-3, 0)

We know that

$$\text{Area} = \int_a^b f(x)dx$$

$$= \int_{-3}^{0} f(x^3 + 3x^2)dx$$

$$= \left|\frac{x^4}{4} + \frac{3x^3}{3}\right|_{-3}^{0}$$

$$= \left|\frac{x^4}{4} + x^3\right|_{-3}^{0}$$

$$= \left(\frac{0}{4} + 0\right) - \left(\frac{(-3)^4}{4} + (-3)^3\right)$$

$$= 0 - \left[\frac{81}{4} - 27\right]$$

$$-\left(\frac{81-108}{4}\right) = -\left(\frac{-27}{4}\right) = \frac{27}{4} \quad \text{square units.}$$

59. **Evaluate the integral** $\int \frac{dx}{\sqrt{x+1}-\sqrt{x}}$ **(C.W) (2 times)**

Sol: $\int \frac{dx}{\sqrt{x+1}-\sqrt{x}}$

$$= \int \left(\frac{1}{\sqrt{x+1}-\sqrt{x}}\right)\left(\frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}\right) dx$$ by rationalization

$$= \int \frac{\sqrt{x+1}+\sqrt{x}}{\left(\sqrt{x+1}\right)^2-\left(\sqrt{x}\right)^2} dx$$

$$= \int \frac{\sqrt{x+1}+\sqrt{x}}{x+1-x} dx$$

$$= \int \left(\sqrt{x+1}+\sqrt{x}\right) dx$$

$$= \int (x+1)^{1/2} dx + \int x^{1/2} dx$$

$$= \frac{(x+1)^{1/2+1}}{\frac{1}{2}+1} + \frac{(x+1)^{1/2+1}}{\frac{1}{2}+1} + c$$

$$= \frac{(x+1)^{3/2}}{\frac{3}{2}} + \frac{(x+1)^{3/2}}{\frac{3}{2}} + c$$

$$= \frac{2}{3}(x+1)^{3/2} + \frac{2}{3}x^{3/2} + c$$ Ans.

60. **Find** $\int \frac{dx}{x(\ln 2x)^3} dx$ **(C.W)**

Sol: $\int \frac{dx}{x(\ln 2x)^3} dx$

$$= \int (\ln 2x)^{-3} \frac{1}{x} dx$$

Let $f(x) = \ln 2x$

$f'(x) = \frac{1}{2x} \times 2$

$$= \int (\ln 2x)^{-3} \left(\frac{1}{2x} \times 2\right) dx$$

$$\int (f(x))^n f'(x) dx = \frac{f(x)^{n+1}}{n+1} + c$$

$$= \int \frac{(\ln 2x)^{-3+1}}{-3+1} + c$$

$$= \frac{(\ln 2x)^{-2}}{-2} + c$$

$$= -\frac{1}{2(\ln 2x)^2} + c$$

61. **Evaluate the integral** $\int x^2 \tan^{-1} x \, dx$ **(H.W)**

Sol: $\int \underset{II}{x^2} \underset{I}{\tan^{-1}} x \, dx$

Integrating by parts

$$= \tan^{-1} x\int x^2dx - \int\left(\frac{d}{dx}\tan^{-1} x\right).\left(\int x^2dx\right)dx$$

$$= \tan^{-1} x\left(\frac{x^3}{3}\right) - \int\frac{1}{1+x^2}\left(\frac{x^3}{3}\right)dx$$

$$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{3}\int\frac{x^3}{1+x^2}\,dx$$

(Improper fraction)

$$\because\ x^2+1\overline{)x^3}\quad \text{quotient } x$$

$$\frac{x^3 \pm x}{-x}$$

$$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{3}\int\left(x - \frac{x}{1+x^2}\right)dx$$

$$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{3}\int x dx + dx\frac{1}{3}\int\frac{1}{1+x^2}dx$$

$$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{3}\left(\frac{x^2}{2}\right) + \frac{1}{3(2)}\int\frac{2x}{1+x^2}dx \qquad \because \int\frac{f'(x)}{f(x)}dx = \ln f(x)+c$$

$$= \frac{x^3}{3}\tan^{-1} x - \frac{x^2}{6} + \frac{1}{6}\ln(1+x^2)+c$$

62. **Evaluate the definite integral** $\int_2^{\sqrt{5}} x\sqrt{x^2-1}\,dx$ **(H.W)**

Sol:

$$\int_2^{\sqrt{5}} x\sqrt{x^2-1}\,dx$$

$$= \int_2^{\sqrt{5}} \left(x^2-1\right)^{1/2} x\,dx$$

Multiply and divide by 2

$$= \frac{1}{2}\int_2^{\sqrt{5}} \left(x^2-1\right)^{1/2}(2x)dx \qquad \because \int f(x)^n f'(x)dx = \frac{(f(x))^{n+1}}{n+1}+c$$

$$= \frac{1}{2}\left[\frac{\left(x^2-1\right)^{1/2+1}}{\frac{1}{2}+1}\right]_2^{\sqrt{5}}$$

$$= \frac{1}{2}\left[\frac{\left(x^2-1\right)^{3/2}}{\frac{3}{2}}\right]_2^{\sqrt{5}} = \frac{1}{3}\left[\left(x^2-1\right)^{3/2}\right]_2^{\sqrt{5}}$$

$$= \frac{1}{3}\left[\left(\left(\sqrt{5}\right)^2-1\right)^{3/2} - \left((2)^2-1\right)^{3/2}\right] = \frac{1}{3}\left[(5-1)^{3/2} - (4-1)^{3/2}\right]$$

$$= \frac{1}{3}\left[4^{3/2} - 3^{3/2}\right] = \frac{1}{3}\left[\left(2^2\right)^{3/2} - 3\sqrt{3}\right] = \frac{1}{3}\left[8 - 3\sqrt{3}\right] = \frac{8}{3} - \sqrt{3} \qquad \textbf{Ans.}$$

63. **Solve the differential equation $\frac{dy}{dx} = \frac{1-x}{y}$** **(H.W)**

Sol: $\frac{dy}{dx} = \frac{1-x}{y}$

Separating the variables

$$y\,dy = (1-x)\,dx$$

Integrating both sides

$$\int y\,dy = \int (1-x)\,dx$$

$$\frac{y^2}{2} = x - \frac{x^2}{2} + c_1$$

Multiply both sides by 2

$$y^2 = 2x - x^2 + 2c_1 \qquad \because c = 2c_1$$

$$y^2 = x(2-x) + c$$

64. **Evaluate : $\int_1^2 \frac{x}{x^2+2}dx$** **(H.W) (2 times)**

Sol: $\int_1^2 \frac{x}{x^2+2}dx = \frac{1}{2}\int_1^2 \frac{2x}{x^2+2}dx$ $\quad\because$ Multiplying and dividing by 2.

$$= \frac{1}{2}\left[\ln(x^2+2)\right]_1^2 \qquad \because \int \frac{f'(x)}{f(x)}dx = \ln f(x) + c$$

$$= \frac{1}{2}\left[(2^2+2) - \ln(1^2+2) - \ln(1^2+2)\right]$$

$$= \frac{1}{2}\left[\ln(6) - \ln(3)\right]$$

$$= \frac{1}{2}\ln\frac{6}{3}$$

$$= \frac{1}{2}\ln 2 = \ln(2)^{1/2} = \ln\sqrt{2}$$

65. **Find the area bounded by the curve $y = 4 - x^2$ and x – axis.** **(C.W)**

Sol: $y = 4 - x^2$

Putting $y = 0$

$$4 - x^2 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

We know that

$$\text{Area} = \int_a^b f(x)dx$$

$$= \int_{-2}^{2}(4-x^2)dx$$

$$= \left|4x - \frac{x^3}{3}\right|_{-2}^{2}$$

$$= \left[4(2) - \frac{(2)^3}{3}\right] - \left[4(-2) - \frac{(-2)^3}{3}\right]$$

$$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$$

$$= \frac{16}{3}+\frac{16}{3} = \frac{32}{3} \text{ square units.}$$

66. **Solve $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$** **(C.W)**

Sol: $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

Separating the variables

$\sec^2 y \tan x\, dy = -\sec^2 x \tan y\, dx$

$$\frac{\sec^2 y}{\tan y}\, dy = -\frac{\sec^2 x}{\tan x}\, dx$$

Integrating both sides

$$\int \frac{\sec^2 y}{\tan y}\, dy = -\int \frac{\sec^2 y}{\tan y}\, dx \qquad \therefore \int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + c$$

$$\ln(\tan y) = -\ln(\tan x) + \ln c$$

$$\ln(\tan y) + \ln(\tan x) = \ln c$$

$$\ln \tan y \tan x = \ln c$$

$$\tan y \tan x = c \qquad \because \ln c = c$$

67. **Evaluate** $\int x(\sqrt{x}+1)dx$ **(H.W)**

Sol: $\int x(\sqrt{x}+1)dx$

$$= \int (x\sqrt{x}+x)dx = \int \left(x^{3/2}+x\right) dx$$

$$= \int x^{3/2}\, dx + \int x\, dx$$

$$= \frac{x^{3/2+1}}{\frac{3}{2}+1} + \frac{x^{1+1}}{1+1} + c = \frac{x^{5/2}}{\frac{5}{2}} + \frac{x^2}{2} + c = \frac{2}{5}x^{5/2} + \frac{x^2}{2} + c$$

68. **Evaluate the integral** $\int x^2 e^{ax} dx$ **(C.W)**

Sol: $\int \underset{I}{x^2}\, \underset{II}{e^{ax}}\, dx$

Integrating by parts

$$= x^2 \int e^{ax} dx - \int \left(\frac{d}{dx}x^2\right)\left(\int e^{ax} dx\right) dx$$

$$= \frac{x^2}{a} e^{ax} - \frac{2}{a}\int \underset{I}{x} \,.\, \underset{II}{e^{ax}}\, dx$$

$$= \frac{x^2}{a} e^{ax} - \frac{2}{a}\left\{x.\frac{e^{ax}}{a} - \int 1\,.\,\frac{e^{ax}}{a} dx\right\}$$

$$= \frac{x^2}{a} e^{ax} - \frac{2x}{a^2} e^{ax} + \frac{2}{a^2}\frac{e^{ax}}{a} + c$$

$$= \frac{x^2}{a} e^{ax} - \frac{2x}{a^2} e^{ax} + \frac{2}{a^3} e^{ax} + c$$

69. **Evaluate** $\int \frac{x+b}{(x^2+2bx+c)^{1/2}}\, dx$ **(H.W)**

Sol:
$$\int \frac{x+b}{(x^2+2bx+c)^{1/2}}\, dx$$

$$= \int (x^2+2bx+c)^{-1/2}(x+b)\, dx$$

Multiplying and dividing by 2

$$=\frac{1}{2}\int\left(x^2+2bx+c\right)^{-1/2}2(x+b)\,dx$$

$$=\frac{1}{2}\frac{(x^2+2bx+c)^{\frac{-1}{2}+1}}{-\frac{1}{2}+1}+c$$

$$=\frac{1}{2}\frac{(x2+2bx+c)^{1/2}}{\frac{1}{2}}+c$$

$$=(x^2+2bx+c)^{\frac{1}{2}}+c_1=\sqrt{x^2+2bx+c}+c_1$$

70. **Evaluate** $\int_0^{\pi/3}\cos^2\theta\sin\theta d\theta$ **(H.W)**

Sol: $=\int_0^{\pi/3}\cos^2\theta\sin\theta d\theta$

Multiplying and dividing by – 1

$$=-\int_0^{\pi/3}\cos^2\theta(-\sin\theta)d\theta \qquad \because \int\left(f(x)\right)^n f'(x)\,dx=\frac{f(x)^{n+1}}{n+1}+c$$

$$=-\left[\frac{\cos^3\theta}{3}\right]_0^{\pi/3}=-\frac{1}{3}\left[\left(\cos\frac{\pi}{3}\right)^3-(\cos 0)^3\right]$$

$$=\frac{-1}{3}\left\{\left(\frac{1}{2}\right)^3-(1)^3\right\}$$

$$=\frac{-1}{3}\left\{\frac{1}{8}-1\right\}$$

$$=\frac{-1}{3}\left[\frac{1-8}{8}\right]$$

$$=\frac{-1}{3}\left(\frac{-7}{8}\right)$$

$$=\frac{7}{24}$$

71. **Evaluate** $\int x^4\ \ell nx\ dx$ **(H.W)**

Sol: $\int \underset{II}{x^4}\ \underset{I}{\ell nx}\ dx$

Integrating by parts

$$=\ell nx\int x^4dx-\int\left(\frac{d}{dx}\ln x\right)\left(\int x^4dx\right)dx$$

$$=\ell nx\left(\frac{x^5}{5}\right)-\int\frac{1}{x}\frac{x^5}{5}dx$$

$$=\frac{x^5}{5}\ell nx-\frac{1}{5}\int x^4dx$$

$$=\frac{x^5}{5}\ell nx-\frac{1}{5}\left(\frac{x^5}{5}\right)+c$$

$$=\frac{x^5}{5}\ell nx-\frac{x^5}{25}+c=\frac{x^5}{5}\left[\ell nx-\frac{1}{5}\right]+c$$

72 Solve the differentiate equation $\sec x + \tan y \frac{dy}{dx} = 0$ **(H.W) (2 times)**

Sol: $\sec x + \tan y \frac{dy}{dx} = 0$

Separating the variables

$$\tan y \frac{dy}{dx} = -\sec x$$

$$\tan y dy = -\sec x dx$$

Integrating on both sides

$$\int \tan y \ dy = -\int \sec x \ dx$$

$$\text{Or } \int \frac{-\sin y}{\cos y} dy = \int \sec x \, dx$$

$$\ell n(\cos y) = \ell n(\sec x + \tan x) + \ell nc$$

$$\ell n(\cos y) = \ell nc(\sec x + \tan x)$$

$$\cos y = c(\sec x + \tan x)$$

73 Evaluate $\int e^x (\cos x + \sin x) dx$ **(C.W)**

Sol: $\int e^x (\cos x + \sin x) dx$

$$\text{Or } = \int e^x (\sin x + \cos x) dx$$

$$\therefore \int e^x [f(x) + f'(x)] dx = e^x f(x) + c$$

$$= e^x \sin x + c$$

74. Evaluate $\int \frac{3 - \cos 2x}{1 + \cos 2x} dx$ **(C.W)**

Sol: $\int \frac{3 - \cos 2x}{1 + \cos 2x} dx$

$$= \int \frac{4 - (1 + \cos 2x)}{1 + \cos 2x} dx = \int \frac{4}{1 + \cos 2x} dx - \int 1 \, dx$$

$$= 4\int \frac{1}{1 + \cos 2x} dx - \int 1 \, dx = 4\int \frac{1}{2\cos^2 x} dx - \int 1 \, dx$$

$$= 2\int Sec^2 x dx - \int 1 \, dx = 2 \tan x - x + c$$

75. Evaluate $\int \frac{Cosx}{Sinx \ln Sinx} dx$ **(H.W)**

Sol: $\int \frac{Cosx}{Sinx \, l\text{n}(Sinx)} dx$

$$= \int \frac{\left(\frac{Cosx}{Sinx}\right)}{l\text{n}(Sinx)} dx \qquad \because \int \frac{f'(x)}{f(x)} dx = l\text{n}|f(x)| + c$$

$$= l\text{n}|l\text{n}(Sinx)| + c$$

76. Evaluate $\int_{-1}^{3} (x^3 + 3x^2) dx$ **(C.W)**

Sol: $= \left[\frac{x^4}{4} + \frac{3x^3}{3}\right]_{-1}^{3}$

$$=\left(\frac{3^4}{4}+3^3\right)-\left(\frac{(-1)^4}{4}+(-1)^3\right)$$

$$=\left(\frac{81}{4}+27\right)-\left(\frac{1}{4}-1\right)=\left(\frac{81+188}{4}\right)-\left(\frac{1-4}{4}\right)$$

$$=\frac{269}{4}+\frac{3}{4}=\frac{269+3}{4}=\frac{272}{4}\cong 68$$

77. **Evaluate** $\int(\ln x)^2\,dx$ **(H.W)**

Sol: $\int(\ln x)^2\,1dx$

Intergrating by parts

$$=(\ln x)^2 x-\int \cancel{x}.2(\ln x).\frac{1}{\cancel{x}}\,dx$$

$$=x(\ln x)^2-2\int(\ln x)\,1dx$$

Again Intergrating by parts

$$=x(\ln x)^2-2\left[\ln x.x-\int \cancel{x}.\frac{1}{\cancel{x}}\,dx\right]$$

$$=x(\ln x)^2-2x\ln x+2x+c$$

78. **Evaluate** $\int\frac{(Sinx+Cos^2x)}{Cos^2x\;Sinx}\,dx$ **(C.W)**

Sol: $\int\left(\frac{Sinx+Cos^2x}{Cos^2x\,Sinx}\right)dx$

$$=\int\left(\frac{\cancel{Sinx}}{Cos^2x\,\cancel{Sinx}}+\frac{\cancel{Cos^2x}}{\cancel{Cos^2x}\,Sinx}\right)dx$$

$$=\int sec^2x\;dx+\int cscx\,dx$$

$$=\tan x+l\text{n}\,|cscx-cotx|+c$$

79. **Find** $\int x\left(\sqrt{x}+1\right)dx$ **(H.W)**

Sol: $\int x\left(\sqrt{x}+1\right)dx$

$$=\int x\left(\sqrt{x}+1\right)dx=\int x^{\frac{3}{2}}dx+\int xdx$$

$$=\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+\frac{x^2}{2}+c=\frac{x^{5/2}}{5/2}+\frac{x^2}{2}+c$$

$$=\frac{2}{5}x^{5/2}+\frac{x^2}{2}+c$$

80. **Evaluate** $\int a^{x^2}xdx$ **(C.W)**

Sol: $\int a^{x^2}xdx$

$$=\frac{1}{2}\int a^{x^2}(2x)\,dx\qquad=\frac{1}{2}\frac{a^{x^2}}{\ln a}+c$$

81. Solve $\dfrac{dy}{dx}=\dfrac{y^2+1}{e^{-x}}$ **(C.W)**

Sol: Given $\dfrac{dy}{dx}=\dfrac{y^2+1}{e^{-x}}$

Seperatign the variables

$$\frac{1}{y^2+1}dy=e^x\,dx$$

Integrating on both sides,

$$\int\frac{1}{1+y^2}\,dx=\int e^x\,dx$$

$$\Rightarrow \tan^{-1}(y)=e^x+c$$

$$\Rightarrow y=\tan\left(e^x+c\right)$$

82. Integrate $\tan^{-1}x$ **w.r.t.** x **(H.W)**

Sol: $\int \tan^{-1}x\,dx$

Integrating by parts.

$$=\tan^{-1}x.x=\int x.\frac{1}{1+x^2}\,dx$$

$$=x\tan^{-1}x-\int\frac{x}{1+x^2}dx=x\tan^{-1}x-\frac{1}{2}\int\frac{2x}{1+x^2}\,dx$$

$$=x\tan^{-1}x-\frac{1}{2}\ln\left|1+x^2\right|+c$$

83. Evaluate $\int(\ln x)\times\dfrac{1}{x}\,dx$ **(H.W)**

Sol: $\int(\ln x)\times\dfrac{1}{x}\,dx$ $\quad\because\int\left(f(x)\right)^n f'(x)\,dx=\dfrac{\left(f(x)\right)^{n+1}}{n+1}+c$

$$=\frac{(\ln x)^{1+1}}{1+1}+c \qquad =\frac{(\ln x)^2}{2}+c$$

LONG QUESTIONS OF CHAPTER-3 ACCORDING TO ALP SMART SYLLABUS-2020

Topic II: Ant-Derivative (Integration):

1. Evaluate $\int\dfrac{\sin x+\cos^3x}{\cos^2x\sin x}dx$ (C.W)
2. Evaluate $\int\dfrac{3-x}{1-x-6x^2}\,dx$ (C.W)
3. Evaluate $\int\dfrac{\sqrt{2}}{\sin x+\cos x}dx$ **(H.W) (3 times)**

Topic III: Substitution Method:

4. Show that $\int\dfrac{dy}{\sqrt{x^2-a^2}}=\ln(x+\sqrt{x^2-a^2})+c$ **(H.W) (6 times)**

Topic VI: Integration by Parts:

5. Evaluate: $\int x^4 . \ln x \, dx$ (H.W) **(2 times)**

6. Show that $\int e^{ax} \sin bx \, dx = \frac{1}{\sqrt{a^2+b^2}} e^{ax} \sin\left(bx - \tan^{-1}\frac{b}{a}\right) + c$ (H.W) **(2 times)**

7. Evaluate $\int \tan^3 x \sec x \, dx$ (H.W) **(2 times)**

Topic VI: Area under the curve:

8. Evaluate the definite integral $\int_2^3 \left(x - \frac{1}{x}\right)^2 dx$ (H.W)

9. Evaluate $\int_{-1}^{1} \left(x + \frac{1}{2}\right)\sqrt{x^2 + x + 1} dx$ (C.W) **(2 times)**

10. Evaluate $\int_1^2 \frac{x^2+1}{x+1} dx$ (C.W)

11. Evaluate : $\int_{-1}^{2} (x + |x|) \, dx$ (C.W) **(2 times)**

12. Evaluate $\int_0^{\frac{\pi}{4}} \frac{\cos x + \sin x}{\cos 2x + 1} dx$ (H.W)

13. Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x - 1}{\cos^2 x} dx$ (H.W) **(2 times)**

14. Evaluate $\int_1^e x \ln x \, dx$ (H.W)

15. Evaluate $\int_0^{\pi/4} \cos^4 t \, dt$ (H.W)

16. Evaluate $\int_0^{\frac{\pi}{0}} \cos^3\theta \, d\theta$ (H.W) **(2 times)**

Topic VII: Solution of Differential Equation:

17. Solve the differential equation $\frac{x^2+1}{y+1} = \frac{x}{y} \cdot \frac{dy}{dx}$ where $x > 0$, $y > 0$. (H.W)**(2 times)**

Chapter-3 (Examples According to ALP Smart Syllabus)

Example 8: (Page#134) Find $\int \frac{dx}{x(\ln 2x)^3}$, $(x > 0)$ **(C.W)**

Sol: Put $\ln 2x = t$, *then*

$$\frac{1}{2x} 2dx = dt \quad or \quad \frac{1}{x} dx = dt$$

$$Thus \int \frac{1}{(\ln 2x)^3} \cdot \frac{1}{x} dx = \int \frac{1}{t^3} . dt = \int t^{-3} dt = \frac{t^{-2}}{-2} + c$$

$$= -\frac{1}{2t^2} + c = -\frac{1}{2(\ln 2x)^2} + c$$

Example10: (Page#134) Evaluate $(i) \int \frac{1}{\sqrt{a^2 - x^2}} dx, (-a < x < a)$ **(C.W)**

$(ii) \int \frac{1}{x\sqrt{a^2 - x^2}} dx, (x - a \ or \ x < -a)$ **where a is positive.**

Sol(i): Let $x = a\sin\theta$ that is,

$x = a\sin\theta$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ then $dx = a\cos\theta d\theta$

Thus

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \int \frac{a\cos\theta d\theta}{\sqrt{a^2 - a^2\sin^2\theta}} = \int \frac{a\cos\theta d\theta}{a\sqrt{1 - a^2\sin^2\theta}} = \int \frac{a\cos\theta d\theta}{a\cos\theta} = \int 1 d\theta = \theta + c$$

$$= \sin^{-1}\left(\frac{x}{a}\right) + c \qquad \left(\because \frac{x}{a} = \sin\theta\right)$$

(ii) Put $x = a\sec\theta$ *i.e.,* $x = a\sec\theta$ *for* $0 < \theta < \frac{\pi}{2}$ *or* $\frac{\pi}{2} < \theta < \pi$

Then $dx = a\sec\theta\tan\theta$

$$\int \frac{dx}{x\sqrt{x^2 - a}} = \int \frac{a\sec\theta\tan\theta d\theta}{a\sec\theta\sqrt{a\sec^2\theta - a^2}} = \int \frac{a\sec\theta\tan\theta dv}{a\sec.a\tan\theta} \left(\because \sqrt{a^2(\sec^2\theta - 1)}\right)$$

$$\frac{1}{a}\int 1 d\theta = \frac{1}{a}\theta + c$$

$$= \frac{1}{a}\sec^{-1}\frac{x}{a} + c \qquad \left(\because \sec\theta = \frac{x}{a}\right)$$

Example 4: (Page#147) Evaluate $\int \frac{7x - 1}{(x-1)^2(x+1)} dx, \quad (x > 1)$ **(C.W)**

Sol: We write

$$\frac{7x - 1}{(x-1)^2(x+1)} dx = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

$$= \frac{2}{x-1} + \frac{3}{(x-1)^2} + \frac{2}{x+1} \qquad \left(\begin{array}{l}\textit{Applying the method of}\\ \textit{Partial Fraction}\end{array}\right)$$

$$\int \frac{7x - 1}{(x-1)^2(x+1)} = \int \left[\frac{2}{x-1} + \frac{3}{(x-1)^2} - \frac{2}{x+1}\right] dx$$

$$= 2\int (x-1)^{-1} 1.dx + 3\int (x-1)^{-2} 1.dx - 2\int (x+1)^{-1} 1.dx$$

$$=2\ln(x-1)+3\frac{(x-1)^{-2+1}}{-2+1}-2\ln(x+1)+c,\ (x>1)$$

$$=2[\ln(x-1)-\ln(x+1)]+3\left[\frac{(x-1)^{-1}}{-1}\right]+c=2\ln\left(\frac{x-1}{x+1}\right)-\frac{3}{x-1}+c$$

Example 8: (Page#149) Evaluate $\int\frac{3}{x(x^3-1)}dx,\qquad x\neq 0, x\neq 1$ **(C.W)**

Sol: **Let** $\frac{3}{x(x^3-1)}=\frac{A}{x}+\frac{B}{x-1}+\frac{Cx+D}{x^2+x+1}$

$$=\frac{-3}{x}+\frac{1}{x-1}+\frac{2x+1}{x^2+x+1}\qquad (By\ the\ method\ of\ partial\ fraction)$$

Let $\int\frac{3}{x(x-1)(x^2+x+1)}dx=\int\left(\frac{-3}{x}+\frac{1}{x-1}+\frac{2x+1}{x^2+x+1}\right)dx$

$$=-3\int(x)^{-1}1.dx+\int(x-1)^{-1}1.dx+\int(x^2+x+1)^{-1}(2x+1)dx$$

$$=-3\ln|x|+\ln|x-1|+\ln(x^2+x+1)+c$$

$$=-3\ln|x|+\ln|x-1|(x^2+x+1)+c=-3\ln|x|+\ln|x^3-1|+c$$

Example 8: Evaluate $\int_{-1}^{e}x\ln x dx$

Sol: **Applying the formula**

$\int f(x)\phi'(x)dx=f(x)\phi(x)-\int\phi(x)f'(x)dx$, *we have*

$$\int(\ln x)xdx=(\ln x)\frac{x^2}{2}-\int\left(\frac{x^2}{2}\right).\frac{1}{x}dx$$

$$\int(\ln x)xdx=\frac{1}{2}x^3\ln x-\frac{1}{2}\int xdx=\frac{1}{2}x^2\ln x-\frac{1}{2}\left(\frac{x^2}{2}\right)+c$$

Thus $\int_1^e x\ln xdx=\left[\frac{1}{2}x^2\ln x-\frac{x^2}{4}\right]_1^e$

$$=\left(\frac{1}{2}e^2\ln e-\frac{e^2}{4}\right)-\left(\frac{1}{2}(1)^2\ln 1-\frac{(1)^2}{4}\right)$$

$$=\left(\frac{e^2}{2}.1-\frac{e^2}{4}\right)-\left(\frac{1}{2}0-\frac{1}{4}\right)$$

$$=\frac{e^2}{4}+\frac{1}{4}$$

OBJECTIVES (MCQ'S) OF CHAPTER-4
ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Coordinate System:

1. The distance of the point (-2, 3) from y-axis is: **(2 times)**
(A) -2 (B) 2 (C) 3 (D) 1

2. Distance between (1, 2) and (2, 1) is: **(8 times)**
(A) $\sqrt{3}$ (B) $\sqrt{5}$ (C) $\sqrt{2}$ (D) $\sqrt{7}$

3. The co-ordinate axes divide the plane into______ equal parts:
(A) 2 (B) 3 (C) 4 (D) 5

4. A vertical line divides a plane into_______ half planes **(3 times)**
(A) Upper and lower (B) Upper and right (C) Left and right (D) Left and lower

5. The ratio in which y-axis divides the line joining (2 , -3) and (-5, 6) is:
(A) 2 : 3 (B) 2 : 5 (C) 1 : 2 (D) 3 : 5

6. The point of concurrency of the medians of a triangle is called: **(2 times)**
(A) In-centre (B) Centroid (C) E-centre (D) Circumcentre

7. The distance of the point (1, 1) from origin is: **(6 times)**
(A) 0 (B) 1 (C) $\sqrt{2}$ (D) 4

8. The distance of the point (3, -7) from the x – axis is.
(a) 7 (b) 3 (c) -3 (d) -7

9. Distance of (-3, 7) from x-axis is.
(a) -3 (b) 3 (c) 7 (d) 10

10. Distance between the points (2,3) and (3,2) is
(A) 2 (B) $\sqrt{2}$ (C) 1 (D) $2\sqrt{5}$

11. Distance of point (1, -2) from y – axis is.
(a) 2 (b) 1 (c) 3 (d) 4

12. Mid point of A (2, 0), B (0, 2) is.
(a) 0, 2 (b) 2. 0 (c) 2, 2 (d) 1, 1

13. P is mid point of AB if P divides AB in the ratio =:
(A) 1:1 (B) 2:2 (C) 1:2 (D) 2:1

14. The distance between two points A (x_1 , y_1) and B (x_2,y_2) is
(A) $(x_2 - x_1)^2 + (y_2 - y_1)^2$
(B) $\sqrt{(x_2 - x_1) + (y_2 - y_1}$
(C) $\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$,
(D) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

15. The point P (x_1, y_1) and the origin are on the same side of ax + by + c = 0 if $ax_1 + by_1 + c$ and c have the: **(3 times)**
(A) c = 0 (B) Same sign (C) Opposite sign (D) Does not possible

16. The distance between the points (0 , 0) and (1 , 2) is: **(2 Times)**
(A) 0 (B) 2 (C) $\sqrt{3}$ (D) $\sqrt{5}$

17. Location of Point P(x , y) for which x = y is in the quadrants:
(A) 1st and 3rd (B) 2nd and 4th (C) 1st and 2nd (D) 3rd and 4th

18. Centroid of a ΔABC is a point that divides each median in the ratio:
(A) 1 : 1 (B) 3 : 2 (C) 2 : 1 (D) 2 : 3

19. The point (3 , -8) lies in the quadrant:

(A) I (B) II (C) III (D) IV

20. **The x-component of a point P(x,y) is called:**
(A) ordinate (B) abscissa (C) coordinate (D) distance from origin

21. **The mid-point divides the line segment in the ratio:**
(A) 2:1 (B) 1:2 (C) 2:3 (D) 1:1

Topic III: Equations of Straight Line:

22. **The slope of line with inclination $90°$ is:** **(4 times)**
(A) 0 (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) Undefined

23. **The perpendicular distance of the line 12x + 5y = 7 from the origin is:** **(3 times)**
(A) 13 (B) $\frac{13}{7}$ (C) $\frac{7}{13}$ (D) $\frac{1}{13}$

24. **The lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ will be perpendicular if**
(A) $\frac{a_1}{b_1}=\frac{a_2}{b_2}$ (B) $a_1a_2=b_1b_2$ (C) $a_1a_2+b_1b_2=0$ (D) $a_1c_1+a_2c_2=0$

25. **If two lines are parallel then:** **(5 times)**
(A) $m_1.m_2=-1$ (B) $m_1.m_2=1$ (C) $m_1+m_2=0$ (D) $m_1-m_2=0$

26. **If α is the inclination of non-vertical line then slope is:** **(2 times)**
(A) $\sin\alpha$ (B) $\cos\alpha$ (C) $\tan\alpha$ (D) $\cot\alpha$

27. **The slope of the line with inclination $30°$ is:** **(4 times)**
(A) zero (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) $\sqrt{3}$

28. **For b > 0 the point (x_1, y_1) is above the line ax + by + c = 0 if:**
(A) $ax_1+by_1+c<0$ (B) $ax_1+by_1+c=0$ (C) $ax_1+by_1+c>0$ (D) $ax_1-by_1-c<0$

29. **In a plane two mutually perpendicular lines are called:** **(3 times)**
(A) Coordinate axes (B) Radii (C) Madians (D) Altitudes

30. **Slope of a line ax + by + c = 0 is:** **(3 times)**
(A) $\frac{a}{b}$ (B) $\frac{b}{a}$ (C) $-\frac{a}{b}$ (D) $\frac{c}{a}$

31. **The perpendicular distance of the line 3x + 4y + 10 = 0 from the origin is:** **(2 times)**
(A) 0 (B) 1 (C) 2 (D) 10

32. **The slope intercept form of straight line is:** **(4 times)**
(A) y = mx + c (B) $\frac{x}{a}+\frac{y}{b}$ (C) $y-y_1=m(x-x_1)$ (D) $x\cos\alpha+y\sin\beta=p$

33. **Equation of the line in form of $y-y_1=m(x-x_1)$ is:** **(4 times)**
(A) Normal form (B) Point slope form (C) Symmetric form (D) Slope intercept form

34. **If A = (-3, 6) and B = (3, 2) then slope of AB is:** **(6 times)**
(A) $\frac{3}{2}$ (B) $-\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $-\frac{3}{2}$

35. **Equation of the line in the form of $x\cos\alpha+y\sin\alpha=p$ is:** **(4 times)**
(A) Symmetric form (B) Intercept form (C) Slope intercept form (D) Normal form

36. **The slope of tangent line to y = f(x) at (x_1, y_1) is :** **(5 times)**
(A) m (B) $\frac{y_2-y_1}{x_2-x_1}$ (C) $f'(x_1)$ (D) $-\frac{dx}{dy}$

37. A linear equation represents a.
(a) Circle (b) Ellipse (c) Parabola (d) Straight line

38. If a = 0 then the line $ax + by + c = 0$
(a) Parallel to x – axis (b) Parallel to y – axis
(c) Perpendicular to x-axis (d) Passes through origin

39. Equation of a line parallel to x-axis is.
(a) $x = 0$ (b) $x = y$ (c) y = constant (d) x = constant

40. Slope of y – axis is: **(2 times)**
(a) 0 (b) 1 (c) - 1 (d) Undefined

41. Slope of line is 1 (one) and angle made by line with x – axis =
(a) 45° (b) 30° (c) 60° (d) 75°

42. Equation of vertical line through (5 , -3) is:-
(A) $x = 5$ (B) $y = -3$ (C) $y = 5$ (D) $x = -3$

43. $\frac{x}{a}+\frac{y}{b} = 1$ is :
(A) Slope intercept form (B) Two intercept form (C) Symmetric form (D) Normal form

44. Slope of line perpendicular to $3x – 4y + 5 = 0$ is:
(A) $\frac{-3}{4}$ (B) $\frac{-4}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$

45. Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2$ = are parallel if:
(A) $\frac{a_1}{a_2} = \frac{-b_1}{b_2}$ (B) $\frac{a_1}{a_2} = \frac{b_2}{b_2}$ (C) $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ (D) None of these

46. Equation of the line bisecting 2nd and 4th quadrant is:
(A) $y = x$ (B) $y = -x$ (C) $y = \frac{x}{\sqrt{2}}$ (D) $y = mx$

47. Equation of line parallel to x-axis =
(A) $x = 0$ (B) $x = y$ (C) $y = a$ (D) $x = a$

48. Length of perpendicular from (0,0) to line $4x-3y-1=0$ equals.
(A) 3 (B) 4 (C) 5 (D) $\frac{1}{5}$

49. Two Intercept from of equation of line is: **(2 times)**
(A) $\frac{x}{a}+\frac{y}{b} = c$ (B) $\frac{x}{a}+\frac{y}{b} = 1$ (C) $\frac{a}{x}+\frac{b}{y} = 0$ (D) $\frac{x}{a}-\frac{y}{b}=0$

50. The slope of the line with inclination 0° is:
(A) 0 (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) $\sqrt{3}$

51. If b = 0 , then the line $ax + by + c = 0$ is parallel to:
(A) y- axis (B) x - axis (C) along x – axis (D) None of these

52. The distance of point P (6, -1) from the line $6x – 4y + 9 = 0$ is:
(A) 49 (B) $\frac{49}{52}$ (C) $\frac{\sqrt{49}}{52}$ (D) $\frac{49}{\sqrt{52}}$

53. If m_1 and m_2 are slopes of two lines then lines are perpendicular if: (3 Times)
(A) $m_1m_2 = 0$ (B) $m_1m_2 + 1 = 0$ (C) $m_1m_2-1=0$ (D) $m_1+m_2 = 0$

54. The two lines $a_1x + b_1y = c_1$; $a_2x + b_2y = c_2$ are parallel if: (2 Times)
(A) $a_1 – a_2 = 0$ (B) $a_1 – b_1 = 0$ (C) $a_1b_1 – a_2b_2 = 0$ (D) $a_1b_2 – a_2b_1 = 0$

55. Equation of horizontal line through (a , b) is:
(A) $y = a$ (B) $y = b$ (C) $x = a$ (D) $x = b$

56. $\frac{y- y_1}{y_2-y_1} = \frac{x- x_1}{x_2-x_1}$ is equation of straight line in:
(A) Two point form (B) Intercept form (C) Symmetric form (D) Point slope form

57. Vertical line has a slope: **(2 times)**
(A) 0 (B) 1 (C) Undefined (D) -1

58. Two lines having slopes m_1 , m_2 are parallel if:
(A) $m1 + m2 = 0$ B) $m_1 – m_2 = 0$ (C) $m1m2 = 1$ (D) $m1m2 = -1$

59. $x\cos a + y\sin a = P$ is the equation of the line:
(A) Slope – Intercept form (B) Two – intercepts from
(C) Point – slope form (D) Normal form

60. Perpendicular distance of point P(6 , -1) from line 3x + 4y + 1 = 0 equals:
(A) 1 (B) 2 (C) 3 (D) 4

61. The perpendicular distance of a line 12x + 5y = 7 from origin is: (2 times)
(A) $\frac{1}{13}$ (B) $\frac{13}{7}$ (C) $\frac{7}{13}$ (D) 13

62. The medians of a triangle are:
(A) Collinear (B) Concurrent (C) Perpendicular (D) Parallel

63. Slope of line parallel to x – axis is: (2 times)
(A) -1 (B) 0 (C) 1 (D) 90°

64. The lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ are perpendicular if and only if:
(A) $a_1b_2 + a_2b_1=0$ (B) $a_1a_2 - b_1b_2=0$ (C) $a_1a_2 + b_1b_2=0$ (D) $a_1b_2 = a_2b_1$

Topic IV: Angle Between Two Lines:

65. The lines l_1, l_2 with slopes m_1, m_2 are parallel if: (2 times)
(A) $m_1m_2 = -1$ (B) $m_1 = m_2$ (C) $m_1 + m_2 = 0$ (D) $m_1m_2 = 1$

66. The point of intersection of medians of a triangle is: (4 times)
(A) Centroid (B) Orthocentre (C) Circumcentre (D) Incentre

67. Point of intersection of two lines x – 2y = -3 and 3x + 2y = 7 will be.
(a) (2,1) (b) (1, 2) (c) (2, 3) (d) (3, 2)

68. Bisectors of angles of a triangle are:
(A) parallel (B) perpendicular (C) concurrent (D) non-concurrent

69. Point of intersection of lines x – 2y + 1 = 0 and 2x – y + 2 = 0 equals:
(A) (1 , 0) (B) (0 , 1) (C) (-1 , 0) (D) (0 , -1)

70. The x – intercept of the line 2x + 3y – 1 = 0 is
(A) 1/2 (B) 2 (C) 3 (D) 1/3

71. Two non-parallel lines intersect each other at:
(A) 1 point (B) 0 point (C) ∞ points (D) 2 points

72. Inclination of line joining two points (-2 , 4) and (5 , 11) equals:
(A) $\pi/3$ (B) $\pi/4$ (C) $\pi/6$ (D) $\pi/2$

73. If α is the inclination of a non-vertical line ℓ , then its slope is:
(A) sin α (B) cos α (C) tan α (D) cot α

74. The point of intersection of altitudes of a triangle is:
(A) centriod (B) orthocenter (C) e-centre (D) circumcentre

Topic V: Homogeneous Equation of Second degree in two variables:

75. The pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ will be orthogonal if:
(A) a + b > 0 (B) a + b = 0 (C) $h^2 - ab < 0$ (D) $h^2 - ab = 0$

76. Two lines represented by $ax^2 + 2hxy + by^2 = 0$ are parallel if:
(A) $h^2 - ab = 0$ (B) $h^2 + ab = 0$ (C) a + b = 0 (D) a – b = 0

77. Pair of lines represented by homogenous equation $ax^2 + 2hxy + by^2 = 0$ through origin will be real and distinct if:
(A) $h^2 > ab$ (B) $h^2 = ab$ (C) $4^2 + ab = 0$ (D) $h^2 < ab$

78. Two lines represented by $ax^2 + 2hxy + by^2 = 0$ are Perpendicular of: (2 Times)
(A) $h^2 - ab = 0$ (B) a + b = 0 (C) $h^2 + ab = 0$ (D) a – b = 0

79. If $ax^2 + 2hxy + by^2 = 0$ is homogeneous equation then pair of lines are real and coincident if:
(A) $h^2 - ab > 0$ (B) $h^2 - ab < 0$ (C) $h^2 - ab = 0$ (D) h + a + b = 0

80- Point of concurrency of medians of a triangle is called:
(A) orthocentre (B) in-centre (C) ex-centre (D) centroiid

81- The lines represented by $ax^2 + 2hxy + by^2 = 0$, are real and coincident if:
(A) $h^2 > ab$ (B) $h^2 = ab$ (C) $h^2 < ab$ (D) $h^2 = a + b$

82- Equation of the line bisecting the first and third quandrnt is:
(A) y = x (B) y = -x (C) y = x + c (D) xy = c

83- **Slope of the line which is perpendicular to the line 2x – 4y + 11 = 0**

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 2 (D) -2

84- **Distance of the point (3, -7) from x – axis is:**

(A) 3 (B) -3 (C) 7 (D) -7

85- **Inclination of a line perpendicular to y – axis is:**

(A) 0^0 (B) 60^0 (C) 30^0 (D) 90^0

86- **The slope of a line which is perpendicular to the line ax + by + c = 0 is:**

(A) $\frac{-a}{b}$ (B) $\frac{b}{a}$ (C) $\frac{-b}{a}$ (D) $\frac{a}{b}$

87- **The point of concurrency of altitude of a triangle is called:**

(A) In – Centre (B) Orthocentre (C) Circumcentre (D) Centroid

88- **The distance of the point (3, 7) from x-axis is:** **(2 times)**

(A) 7 (B) 3 (C) -3 (D) -7

89- **If the distance of the point (5, x) from x-axis is 3, then x =:**

(A) 7 (B) 5 (C) 3 (D) -5

90- **If (3, 5) is the midpoint of (5, y), (x, 7) then x = ? and y = ?:**

(A) y = 1, x = 1 (B) y = -4, x = -3 (C) y = 3, x = 1 (D) y = -2, x = -5

91- **The slope of line with inclination 60^0 is:**

(A) 0 (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) $\sqrt{3}$

92- **If α is the inclination of the line ℓ then $\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\sin\alpha}=r$ (say) is called:**

(A) Point slope form (B) Normal form (C) Symmetric form (D) Intercept form

93- **If α is the inclination of a line "ℓ" then it must be true that:**

(A) $0\le\alpha<\frac{\pi}{2}$ (B) $\frac{\pi}{2}\le\alpha<\pi$ (C) $0\le\alpha<\pi$ (D) $0\le\alpha<2\pi$

94- **The perpendicular distance of line 3x + 4y - 10 = 0 from the origin is:**

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) 2

95- **Two lines represented by $ax^2 + 2bxy + by^2 = 0$ are orthogonal if:**

(A) a – b = 0 (B) a + b = 0 (C) a + b > 0 (D) a + b < 0

96- **The point of intersection of medians of a triangle is called:**

(A) Circumcenter (B) Orthocenter (C) Centroid (D) In-center

97- **Distance of the points (2, 3) from y – axis is:**

(A) 2 (B) 3 (C) 5 (D) $\sqrt{15}$

98. **Coordinates of mid – point of A(-1, 4), B (6, 2)**

(a) (-7, 2), (b) (7, -2) (c) $\left(\frac{5}{2},3\right)$ (d) $\left(3,\frac{5}{2}\right)$

99. **If m_1, m_2 are slopes of perpendicular lines, then m_1, m_2 =**

(a) 0 (b) – 1 (c) 1 (d) Undefined

100. **If a line meets x and y axes at 2, 3 units, then its equation is:**

(a) 2x + 3y = 0 (b) 3x + 2y = 0 (c) $\frac{x}{2}+\frac{y}{3}=0$ (d) $\frac{x}{2}+\frac{y}{3}=1$

101. **Slope of tangent to the curve $x^2-y^2-12=0$ at point $(4,2)$ will be equal to:**

(a) 4 (b) $1/4$ (c) 2 (d) $\frac{1}{2}$

102. **If the straight lines represented by $ax^2+2hxy+by^2=0$ are perpendicular, then**

(a) $h^2-ab=0$ (b) $h^2+ab=0$ (c) $a+b=0$ (d) $a-b=0$

103. **Equation of straight line passing through $(0,0)$ and parallel to the line with slop 2 will be**

(a) $x=\frac{2}{3}y$ (b) $x=y$ (c) $y=\frac{1}{2}x$ (d) $y=2x$

104. The lines through origin represented by $ax^2+2hxy+by^2=0$ are clubcudent if:

(a) $h^2=ab$ (b) $h^2+ab=0$ (c) $h^2-ab>0$ (d) $h^2-ab<0$

105. If a line "l" is parallel to x-axis then inclination=

(a) 90^0 (b) 0^0 (c) 30^0 (d) 45^0

106. If the line $\frac{x}{a}+\frac{y}{3}=1$ is parallel to the line $3x-2y+4=0$, then value of 'a' equals

(a) -2 (b) 2 (c) 3 (d) 4

107. Equation of a non vertical line with slope m and y intercept zero is

(a) $y=x$ (b) $y=mx$ (c). $y=mx+c$ (d) $y=0$

108. The vertices of a triangle are $(a,b-c),(b,c-a),(c,a-b)$ then its centroid is

(a) $\left(0,\frac{a+b+c}{3}\right)$ (b) $\left(0,\frac{a-b-c}{3}\right)$ (c) $(0,0)$ (d) $\left(\frac{a+b+c}{3},0\right)$

109. The point of concurrency of altitudes of a triangle is called

(a) centroid (b) orthocenter (c) in centre (d) circum centre

ANSWERS TO THE MULTIPLE CHIOCE QUESTIONS

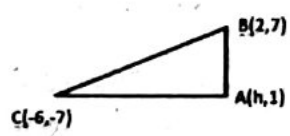
1	2	3	4	5	6	7	8	9	10	11	12	13	14
b	c	c	c	b	b	c	a	c	b	b	d	a	d
15	16	17	18	19	20	21	22	23	24	25	26	27	28
b	d	a	c	d	b	d	d	c	c	d	c	b	c
29	30	31	32	33	34	35	36	37	38	39	40	41	42
a	c	c	a	b	b	d	c	d	a	c	d	a	a
43	44	45	46	47	48	49	50	51	52	53	54	55	56
B	b	c	b	c	d	b	a	a	d	b	d	a	a
57	58	59	60	61	62	63	64	65	66	67	68	69	70
c	b	d	c	c	b	b	c	b	a	b	c	c	a
71	72	73	74	75	76	77	78	79	80	81	82	83	84
a	b	c	b	b	a	A	b	c	d	b	a	d	C
85	86	87	88	89	90	91	92	93	94	95	96	97	98
a	b	b	a	c	c	d	c	c	d	b	c	a	C
99	100	101	102	103	104	105	106	107	108	109			
b	d	d	c	d	a	b	a	b	d	b			

SHORT QUESTIONS OF CHAPTER-4 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Coordinate System:

1. Find h such that points A(h, 1), B(2, 7), C(-6, -7) are vertives of a right triangle with right angle at vertex A. (2 times)

Sol: As ΔABC is a right Δ with right angle at vertex A so



$$|BC|^2=|CA|^2+|AB|^2$$

$$(-7-7)^2+(-6-2)^2=\left[(h+6)^2+(1+7)^2\right]+\left[(h-2)^2+(1-7)^2\right]$$

$$196+\cancel{64}=h^2+36+12h+\cancel{64}+h^2+4-4h+36$$

$$196-76=2h^2+8h$$

$$120 = 2h^2 + 8h$$
$$2h^2 + 8h - 120 = 0$$
$$h^2 + 4h - 60 = 0$$
$$h^2 + 10h - 6h - 60 = 0$$
$$h(h+10) - 6(h+10) = 0$$
$$(h+10)(h-6) = 0$$
$$h+10 = 0 \qquad h-6 = 0$$
$$h = -10 \qquad h = 6$$

2. The points A (-5 , -2) and B(5 , -4) are ends of a diameter of a circle. Find its centre and Radius. (6 times)

Sol: Given points are A (-5 , -2) & B (5 , -4) Since A & B are end points of diameter of circle. So centre will be the mid point of line segment AB.

Let C(x , y) be the centre of circle the

$$C(x, y) = \left(\frac{-5+5}{2}, \frac{-2-4}{2}\right)$$
$$C(x, y) = \left(\frac{0}{2}, \frac{-6}{2}\right) = (0, -3)$$

Let r be the radius of circle then

$$|BC| = r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$r = \sqrt{(5-0)^2 + (-4+3)^2}$$
$$r = \sqrt{25+1} = \sqrt{26}$$

A(-5,-2) 

B (5,-4)

3. Show that points A (0 , 2) , B $(\sqrt{3}, -1)$ and C(0 , -2) are vertices of a right triangle. (3 times)

Sol: Given points are A(0 , 2) , B $(\sqrt{3}$, -1) and C (0 , -2)

Now $|AB| = \sqrt{(0-\sqrt{3})^2 + (2+1)^2}$

$|AB| = \sqrt{3+9} = \sqrt{12}$

$|BC| = \sqrt{(\sqrt{3}-0)^2 + (-1+2)^2}$

$|BC| = \sqrt{3+1} = \sqrt{4}$

$|AC| = \sqrt{(0-0)^2 + (2+2)^2}$

$|AC| = \sqrt{0+16} = \sqrt{16}$

Now $|AB|^2 + |BC|^2 = |AC|^2$

$(\sqrt{12})^2 + (\sqrt{4})^2 = (\sqrt{16})^2$

$12 + 4 = 6$

$16 = 16$

Pythagoras theorem is satisfied

Hence A (0 , 2) , B$(\sqrt{3}$, -1) & C (0 , -2) are the vertices of a right triangle with right angle at B.

4. Find the mid point of the line joining the two points A(3 , 1) , B(-2 , -4).

Hence A (0 , 2) , B($\sqrt{3}$, -1) & C (0 , -2) are the vertices of a right triangle with right angle at B.

4. Find the mid point of the line joining the two points A(3 , 1) , B(-2 , -4).

Sol: Given points are

A (x_1, y_1) = A (3 , 1)

B (x_2 , y_2) = B(-2 , -4)

Let M be the midpoint of line segment AB then

M (x , y) = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

M (x , y)= $\left(\frac{3-2}{2}, \frac{1-4}{2}\right)$

M (x , y)= $\left(\frac{1}{2}, -\frac{3}{2}\right)$

Hence M(x , y) = $\left(\frac{1}{2}, -\frac{3}{2}\right)$

5. Find the mid-point of the line joining the two points A(-8, 3), B(2, -1).

Sol **Given Points** A (-8, 3), B (2, -1)

Mid point of the line joining the two points A & B is

$$M(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M(x,y) = \left(\frac{-8+2}{2}, \frac{3-1}{2}\right)$$

$$M(x,y) = \left(\frac{-6}{2}, \frac{2}{2}\right)$$

$$M(x,y) = (-3, 1)$$

Topic II: Translation and Rotation of Axes:

6. The xy coordinate axes are rotated through angle $\theta = 30°$ and axes are OX and OY. Find (X, Y) coordinates of P with P(x, y) = (-5, 3)

Sol: P(x, y) = (-5, 3)

$P(X,Y) = ?$

$\theta = 30°$

We know that

$X = x\cos\theta + y\sin\theta$

$X = -5\cos 30 + 3\sin 30$

$X = -5\left(\frac{\sqrt{3}}{2}\right) + 3\left(\frac{1}{2}\right) = \frac{-5\sqrt{3}+3}{2} = \frac{3-5\sqrt{3}}{2}$

$Y = y\cos\theta - x\sin\theta$

$Y = 3\cos 30 + 5\sin 30$

$Y = 3\left(\frac{\sqrt{3}}{2}\right) + 5\left(\frac{1}{2}\right) = \frac{3\sqrt{3}+5}{2}$

So, P(x, y) = P $\left(\frac{3-5\sqrt{3}}{2}, \frac{3\sqrt{3}+5}{2}\right)$

7. If (x , y) co-ordinates of a point are (-2 , 6). Find (X , Y) trSolformed co-ordinates if new origin is O(-3 , 2)

The co-ordinates of P referred to translated axis O'x O' & O'y are

$X = x - h \quad = -2 + 3 = 1$

$Y = y - k \quad = 6 - 2 = 4$

Hence P (X ,Y) = P(1 , 4)

8. The xy coordinate axes are rotated about the origin, through and angle of 45°. The new axes OX and OY. Find the (X , Y) coordinates of P (5 , 3) (2 times)

Sol: Given point of $P(x_1, y_1) = P(5, 3)$ angle of rotation is $\theta = 45°$

Suppose P(X , Y) be the coordinates of Preferred to XY – Co-ordinate system, them

$X = x\cos\theta + y\sin\theta$, $Y = -x\sin\theta + y\cos\theta$

$X = 5\cos 45° + 3\sin 45°$, $Y = -5\sin 45° + 3\cos 45°$

$X = 5\left(\frac{1}{\sqrt{2}}\right) + 3\left(\frac{1}{\sqrt{2}}\right)$, $Y = -5\left(\frac{1}{\sqrt{2}}\right) + 3\left(\frac{1}{\sqrt{2}}\right)$

$X = \frac{5}{\sqrt{2}} + \frac{3}{\sqrt{2}}$, $Y = \frac{-5}{\sqrt{2}} + \frac{3}{\sqrt{2}}$

$X = \frac{8}{\sqrt{2}}$, $Y = \frac{-5+3}{\sqrt{2}}$

$X = \frac{4+2}{\sqrt{2}}$, $Y = \frac{-2}{\sqrt{2}} = -\frac{(\sqrt{2})^2}{\sqrt{2}}$

$X = 4\sqrt{2}$, $Y = -\sqrt{2}$

So $(X,Y) = \left(4\sqrt{2}, -\sqrt{2}\right)$ are required co-ordiantes of P.

Topic III: Equations of Straight Line:

9. Find whether points (5, 8) lies above or below the line $2x - 3y + 6 = 0$ (2 times)

Sol: Given equation is

$2x - 3y + 6 = 0$

Make coefficient of y + ve

$3y - 2x - 6 = 0$ .... (i)

Now put (5, 8) in L.H.S of equation (i)

$L.H.S = 3y - 2x - 6$

$= 3(8) - 2(5) - 6$

$= 24 - 20 - 6$

$= 8 > 0$

So point above the line.

10. Find the distance between parallel lines $2x - 5y + 13 = 0$; $2x + 5y - 6 = 0$ (2 times)

Sol: Parallel lines are:

$2x - 5y + 13 = 0$ .... (i)

$-2x + 5y - 6 = 0$ .... (ii)

Put x = 0 in (i)

$5y + 13 = 0$

$y = \frac{13}{5}$

So point $P\left(0, \frac{13}{5}\right)$

Lines on line (i)

Now distance 'd' of point $P\left(0, \frac{13}{5}\right)$ from line

$-2x + 5y - 6 = 0$

$$d = \frac{\left|-2(0) + 5\left(\frac{13}{5}\right) - 6\right|}{\sqrt{(-2)^2 + (5)^2}}$$

$d=\frac{|13-6|}{\sqrt{4+25}}=\frac{7}{29}$

Required distance.

11. **Find k so that the joining A(7, 3), B(k, -6) and the line joining C(-4, 5), D(-6, 4) are parallel.** **(2 times)**

Sol: Slope of line $\overline{AB}=\frac{-6-3}{k-7}=\frac{-9}{k-7}$

Slope of line $\overline{CD}=\frac{4-5}{-6+4}=\frac{-1}{-2}=\frac{1}{2}$

As given lines are parallel so,

Slope of AB = Slope of CD

$\frac{-9}{k-7}=\frac{1}{2}$

-18 = k − 7

-18 + 7 = k

k = -11

12. **Whether the lines 2x + y − 3 = 0 and 4x + 2y + 5 = 0 are perpendicular or not?**

Sol: Suppose given lines are

$l_1 = 2x + y - 3 = 0 \Rightarrow y = -2x + 3$

$l_2 = 4x + 2y + 5 = 0 \Rightarrow y = -2x - \frac{5}{2}$

Slope of $l_1 = m_1 = -\frac{2}{1} = -2$

Slope of $l_2 = m_2 = -\frac{4}{2} = -2$

So l_1 & l_2 are not a perpendicular

13. **Find the distance from the point P(6, -1) to the line 6x − 4y + 9 = 0. (3 times)**

Sol: Given point $P(x_1, y_1) = P(6, -1)$ and Given line is $6x - 4y + 9 = 0$

Let d be the required distance then

$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$=\frac{|6(6)-4(-1)+9|}{\sqrt{(6)^2+(-4)^2}} \qquad =\frac{|36+4+9|}{\sqrt{36+16}}$

$d=\frac{|49|}{\sqrt{52}}$

$d=\frac{49}{\sqrt{52}}$

14. **Find an equation of the vertical line through (-5, 3)** **(3 times)**

Sol: Given $P(x_1,y_1)=P(-5,3)$

$\because$ line is vertical

$\because \theta=90^0$

So $m=\tan\theta=\tan 90^0=\infty$

Now

$y-y_1=m(x-x_1)$

$y-3=\infty(x+5)$

$\frac{y-3}{\infty}=x+5$

$0=x+5$

OR $x = -5$

15. Find K so that the line joining A (7 , 3) , B(K , -6) and line joining C (-4 , 5) , D(-6 , 4) are perpendicular. **(2 times)**

Sol: Given points are A (7 , 3) , B(K , -6) and C (-4 , 5) , D(-6 , 4)

Hence Slope of AB = $m_1 = \frac{-6-3}{k-7} = \frac{-9}{k-7} = \frac{9}{7-k}$

Slope of CD = $m_2 = \frac{4-5}{-6+4} = \frac{-1}{-2} = 1/2$

Given AB ⊥ CD.

So $m_1 m_2 = -1$

$\left(\frac{9}{7-k}\right)\left(\frac{1}{2}\right) = -1$

$\frac{9}{14-2k} = -1$

$-14 + 2k = 9$

$2k = 9 + 14$

$2k = 23$

$k = \frac{23}{2}$

16. Find equation of line through (-4 , -6) and perpendicular to the line having slope = $\frac{-3}{2}$. **(7 times)**

Sol: Let l be the required line with point $P(x_1, y_1) = P(-4, -6)$

Since l is perpendicular to given line with slope is $-\frac{3}{2}$

So Slope of l is $m = \frac{2}{3}$

Hence equation of l is

$y - y_1 = m(x - x_1)$

$y + 6 = \frac{2}{3}(x + 4)$

$3y + 18 = 2x + 8$

$2x - 3y + 8 - 18 = 0$

$2x - 3y - 10 = 0$

17. Find an equation of line through A(-6 , 5) having slope 7. **(5 times)**

Sol: Let l be the required line

Here $P(x_1, y_1) = P(-6, 5)$

Slope = m = 7

The equation of l is

$y - y_1 = m(x - x_1)$

$y - 5 = 7(x + 6)$

$y - 5 = 7x + 42$

$7x + 42 = y - 5$

$7x - y + 42 + 5 = 0$

$7x - y + 47 = 0$

18. By means of slope, show the points lie on the same line A(-1, -3), B(1, 5), C(2, 9). **(3 times)**

Sol Let A(-1, -3) , B (1, 5) and C(2, 9) are the given points.

Now Slope of $\overline{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-3)}{1-(-1)} = \frac{5+3}{1+1} = \frac{8}{2} = 4$

Slope of $\overline{BC} = \frac{y_2 - y_1}{x_2 - x_1} == \frac{9-5}{2-1} = \frac{4}{1} = 4$

Since Slope of $\overline{AB}$ = Slope of $\overline{BC}$

Thus A, B & C lie on the same line.

19. Convert the equation into two intercepts form $4x + 7y - 2 = 0$ (4 times)

Sol **Given equation** $4x + 7y - 2 = 0$

$$4x + 7y = 2$$

Dividing by 2 on both sides

$$\frac{4x}{2} + \frac{7y}{2} = \frac{2}{2}$$

$$2x + \frac{7y}{2} = 1$$

$$\frac{x}{1/2} + \frac{y}{2/7} = 1$$

Which is required two intercept form with

$$x - \text{intercept} = \frac{1}{2}$$

$$y - \text{intercept} = \frac{2}{7}$$

20. Find whether the point (5, 8) lies above or below the line $2x - 3y + 6 = 0$. (3 times)

Sol Given equation $2x - 3y + 6 = 0$ ____(1) at point (5, 8)

Since y is always positive So multiply eq (1) by -1 on both sides.

$$-2x + 3y - 6 = 0 \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

Put (5, 8) in eq (2)

$= -2(5) + 3(8) - 6$

$= -10 + 24 - 6$

$= 8 > 0$

Hence point (5, 8) lies above the given line.

Topic V: Homogeneous Equation of Second degree in two variables:

21. Find lines represented by $3x^2 + 7xy + 2y^2 = 0$. (3 times)

Sol: $3x^2 + 7xy + 2y^2 = 0$

$3x^2 + 6xy + xy + 2y^2 = 0$

$3x(x + 2y) + y(x + 2y) = 0$

$(3x + y)(x + 2y) = 0$

So required lines are

$3x + y = 0$ & $x + 2y = 0$

22. Find an equation of each of the lines represented by $20x^2 + 17xy - 24y^2 = 0$ (2 times)

Sol **Given equation** $20x^2 + 17xy - 24y^2 = 0$

$20x^2 + 32xy - 15xy - 24y^2 = 0$

$4x(5x + 8y) - 3y(5x + 8y) = 0$

$(4x - 3y)(5x + 8y) = 0$

$4x - 3y = 0$ or $5x + 8y = 0$

Hence required pair of lines

$4x - 3y = 0$

and $5x + 8y = 0$

23 Find the area of region bounded by the triangle with vertices (a, b + c), (a, b – c) and (-a, c)

Sol: Let A(a, b + c), B(a, b – c) and C(-a, c) be the vertices of triangle ABC

Now area of triangle is:

$$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ a & b-c & 1 \\ -a & c & 1 \end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ 0 & 2c & 0 \\ -a & c & 1 \end{vmatrix} by\ R_2-R_1$$

Expanding by second row

$$==\frac{1}{2}[-2c(a+a)]$$

$$=-c(2a)$$

$$=-c(2a)$$

$$=-2ac$$ ∴ Area in always Positive

$$\Delta=2ac$$ square units.

24 **Show that lines 4x – 3y – 8 = 0 , 3x – 4y – 6 = 0 and x – y – 2 = 0 are concurrent.** **(2 times)**

Sol: Let $\ell_1: 4x-3y-8=0$

$\ell_2: 3x-4y-6=0$

$\ell_3: x-y-2=0$

lines ℓ_1,ℓ_2,ℓ_3 are concurrent

if $$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}=0$$

⇒ 4(8 – 6) + 3 (-6 + 6) -8(-3 + 4) = 0
⇒ 4(2) + 3 (0) -8(1) = 0
⇒ 8 – 8 = 0
⇒ 0 = 0

So lines are concurrent

25 **Find volume of "P" such that lines 2x – 3y – 1 = 0 , 3x – y – 5 = 0 and 3x.Py + 8 = 0 meet at a point $(C.W)$.**

Sol: Let $\ell_1: 2x-3y-1=0$

$\ell_2: 3x-y-5=0$

$\ell_3: 3x-Py+8=0$

∵ meet at a point

∵ lines are concurrent

so $$\begin{vmatrix} 2 & -3 & -1 \\ 3 & -1 & -5 \\ 3 & p & 8 \end{vmatrix}=0$$

Expanding by R_1

$$2(-8+5P)+3(24+15)-1(3P+3)=0$$
$$-16+10P+72+45-3P-3=0$$
$$7P+98=0$$
$$7P=-98$$
$$P=-\frac{98}{7}$$
$$P=-14$$

26. Find distance from the point $P(6,-1)$ to the line $6x-4y+9=0$

Sol: Given $P(x_1,y_1)=P(6,-1)$ and $ax+by+c=0=6x-4y+9$

using formula

$$\perp \text{ distance } = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$\perp \text{ distance } = \frac{|6(6)+(-4)(-1)+9|}{\sqrt{(6)^2+(-9)^2}}$$

$$\perp \text{ distance } = \frac{|36+4+9|}{\sqrt{49}}$$

$$\perp \text{ distance } = \frac{49}{\sqrt{49}} = \frac{(\sqrt{49})^2}{\sqrt{49}} = \sqrt{49}$$

27. Show that the lines $2x+y-3=0$ & $4x+2y+5=0$ are parallel.

Sol: Given $l_1: 2x+y-3=0$

And $l_2: 4x+2y+5=0$

So $m_1=-\frac{a}{b}=-\frac{2}{1}=-2$

and $m_2=\frac{-4}{2}=-2$

$\because m_1=m_2$

i.e $-2=-2$

so lines are II.

LONG QUESTIONS OF CHAPTER-4 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Coordinate System:

1. Find 'h' such that the points A(h , 1) ; B (2 , 7) and C (-6 , -7) are the vertices of a right triangle with right angle as the vertex A. **(H.W) (2 times)**
2. Find h such that the point A$(\sqrt{3},-1)$, $B(0,2)$ and C(4 , -2) are vertices of right triangle with right angle at the vertex A. **(H.W)**

Topic II: Equations of Straight Line:

3. Find an equation of the line through (5, -8) and perpendicular to the join of A(-15, -8) , B (10, 7). (H.W) (2 tir

4. Find the conditions that the lines $y=m_1x+c_1\,;y=m_2x+c_2\,;y=m_3x+c_3$ concurrent. (H.W) (3 times)

5. Find equation of two parallel lines perpendicular to 2x − y + 3 = 0 such that the product of the x and y intercept of each is 3. (H.W) (8 times)

6. Find distance between 3x − 4y + 3 = 0 and 3x − 4y + 7 = 0. Also find equation of parallel line lying midway between them. (H.W)

7. By means of slopes, show that the points lie on the same line (-1 , -3) , (1,5) , (2,9) (H.W) (2 times)

8. One vertex of a parallelogram is (1 , 4), the diagonals intersect at (2 , 1) and the sides has slope 1 and $-\frac{1}{7}$. Find other three vertices. (C.W)

Topic IV: Angle Between Two Lines:

9. Find an equation of line through the intersection of the lines x − y − 4 = 0 and 7x + y + 20 = 0 and parallel to the line 6x + y − 14 = 0. (H.W) (2 times)

10. Determine the value of P such that the lines 2x − 3y − 1 = 0, 3x − y − 5 = 0 and 3x + py + 8 = 0 meet at a point. (C.W) (2 times)

11. Find the interior angles of the triangle whose vertices are A (6, 1), B (2, 7), C (- 6,- 7) (H.W)

12. Find the angles of the triangle whose vertices are A(-5, 4), B(-2, -1), C(7, -5)

13. Find an equation of the line through the point of intersection of the lines ℓ_1:3x − 4y -10 = 0 , ℓ_2 : x + 2y − 10 = 0 and perpendicular to the line ℓ : 3x − 4y + 1 = 0 (H.W)

14. Find an equation of the line through the intersection of the lines x + 2y + 3 = 0 , 3x + 4y + 7 = 0 and making equal intercepts on the axes. (H.W)

Topic V: Homogeneous Equation of Second degree in two variables:

15. Find lines represented by $2x^2+3xy-5y^2=0$, also find measure of angle between them. (H.W) (2 times)

Chapter-4 (Examples According to ALP Smart Syllabus)

Example 3:(Pag#183) Find the coordinates of the point that divides the join of A(-63) and B(5,-2) in the ration 2:3. (i) internally (ii) externally (C.W)

Sol(i): Here $k_1=2,\ k_2=3, x_1=-6, x_2=5$

By the formula, we have

$$x=\frac{2\times5+3\times(-6)}{2+3}=\frac{-8}{5}\ and\ y=\frac{2(-2)+3(3)}{2+3}=1$$

Coordinates of the required point are $\left(\frac{-8}{5},1\right)$

(ii) In this case

$$x=\frac{2\times5-3\times(-6)}{2-3}=-28\ and\ y=\frac{2(-2)-3(3)}{2+3}=13$$

Coordinates of the required point are $(-28,13)$

Example 3: (Pag#189) The xy-coordinate axes re rotated about the origin through and angle of 30°. If the xy-coordinates of a point are (5,7), find its XY-coordinates, where OX and OY are the axes obtained after rotation. (C.W)

Sol: Let (X,Y) be the coordinates of P referred to the XY-axes. Here $\theta = 30°$.

From equations (3) above, we have

X = 5cos30° + 7 sin30° and Y = -5sin 30° + 7cos30°

$$\Rightarrow X = \frac{5\sqrt{3}}{2} + \frac{7}{2} \quad \text{and } Y = \frac{-5}{2} + \frac{7\sqrt{3}}{2}$$

i.e., $(X,Y) = \left(\frac{5\sqrt{3}}{2} + \frac{7}{2}, \frac{-5}{2} + \frac{7\sqrt{3}}{2}\right)$ are the required coordinates.

Example 11: (Pag#203) Find the distance between the parallel lines. (C.W)

2x + y + 2 = 0 (1)

6x +3y - 8 = 0 (2)

Sketch the lines. Also find an equation of the line parallel to the given lines and lying midway between them.

Sol: We first convert both the lines into normal form (1) can be written as 2x + y = - 2

Dividing both sides by $-\sqrt{4+1}$, we have

$$\frac{-2}{\sqrt{5}}x + \frac{-y}{\sqrt{5}} = \frac{2}{\sqrt{5}}$$

Which is normal form of (1). Normal form of (2) is

$$\frac{6x}{\sqrt{45}} + \frac{3y}{\sqrt{45}} = \frac{8}{\sqrt{45}}$$

i.e., $\frac{2x}{\sqrt{5}} + \frac{y}{\sqrt{5}} = \frac{8}{3\sqrt{45}}$

Length of the perpendicular from (0,0) to the line (1) is $\frac{2}{\sqrt{5}}$, [From (3)]

Similarly, length of the perpendicular from (0,0) to the line (2) is $\frac{8}{3\sqrt{45}}$ [From (4)

From the graphs of the lines it is clear that the lines are on opposite sides of the origin, so the distance between them equals the sum of the two perpendicular lengths.

i.e., Required distance $= \frac{2x}{\sqrt{5}} + \frac{y}{\sqrt{5}} = \frac{8}{3\sqrt{45}}$

The line parallel to the given lines lying midway between them is such that length of the perpendicular from O to the line $= \frac{8}{3\sqrt{45}} - \frac{7}{3\sqrt{5}}\left(or\ \frac{7}{3\sqrt{5}} - \frac{2}{\sqrt{5}}\right) = \frac{1}{3\sqrt{5}}$

Required line is $\frac{2x}{\sqrt{5}}+\frac{y}{\sqrt{5}}=\frac{1}{3\sqrt{5}}$

Example 4: (Pag#213) Find the distance between the parallel lines. **(C.W)**

$$l_1: 2x-5y+13=0$$

$$l_2: 2x-5y+6=0$$

Sol: First find any point on one of the lines,.

Say l_1 if x = 1 lies on l_1 then y = 3 and the point (1,3) lies on it. The distance d from (1,3) to

$$l_2 \text{ is } d=\frac{|2(1)-5(3)+6|}{\sqrt{2^2+5^2}}=\frac{|2-15+6|}{\sqrt{4+25}}=\frac{7}{\sqrt{29}}$$

This distance between the parallel lines is $\frac{7}{\sqrt{29}}$.

Example 1: Find an equation of each of the lines represented by. $20x^2+17xy-24y^2=0$

Sol: The equation may be written as

$$24\left(\frac{x}{y}\right)^2-17\left(\frac{x}{y}\right)-20=0$$

$$\Rightarrow \frac{x}{y}=\frac{17\pm\sqrt{289+1920}}{48}=\frac{17\pm 47}{48}=\frac{4}{3},\frac{-5}{8}$$

$$\Rightarrow y=\frac{3}{4}x \quad and \quad y=\frac{-5}{8}x$$

$$\Rightarrow 4x-3y=0 \quad and \quad 5x+8y=0$$

Example 3: (Pag#228) Find a joint equation of the straight lines through the origin perpendicular to the lines represented by.

Sol: (1) may be written as = 0

(x – 2y) (x+3y) = 0

Thus lines represented by (1) are

x – 2y = 0 (2)

x + 3y = 0 (3)

The line through (0,0) and perpendicular to (2) is

y = -2x or y + 2x = 0 (4)

Similarly, the line through (0,0) and perpendicular to (3) is

y = 3x or y - 3x = 0 (5)

Joint equation of the lines (4) and (5) is

$(y+2x)(y-3x)=0$ or $y^2-xy-6x^2=0$

OBJECTIVES (MCQ'S) OF CHAPTER-5
ACCORDING TO ALP SMART SYLLABUS-2020

1. An expression involving any one of the symbol $<, \leq, \geq, >$ is called:
(A) An equation (B) Non-inequality (C) Identity (D) Inequality

2. The non-negative inequalities are called: (4 times)
(A) Parameters (B) Constants (C) Decision variables (D) Vertices

3. The feasible solution which maximize or minimize the objective function is called the: (1 time)
(A) Feasible region (B) Optimal solution
(C) Converx region (D) Feasible solution set

4. $ax + by < c$ is linear inequality in variables: (3 times)
(A) 2 (B) 3 (C) 1 (D) 0

5. The solution of $ax + by < c$ is: (5 times)
(A) Closed half plane (B) Open half plane (C) Parabola (D) Hyperbola

6. (2, 1) is the solution of inequality (6 times)
(A) $x + y < 5$ (B) $x + y > 5$ (C) $x + y = 5$ (D) $x - y > 5$

7. (0, 1) is not solution if inequality (4 times)
(A) $7x + 2y < 8$ (B) $x - 3y < 0$ (C) $3x + 5y < 7$ (D) $3x + 5y \leq 3$

8. (1, 2) is the solution of: (3 times)
(A) $x + y > 0$ (B) $x + y < 0$ (C) $x + y = 0$ (D) $x - y = 1$

9. Which one is a solution of inequality $2x + 3y < 0$: (2 times)
(A) (-1, -2) (B) (1, +2) (C) (2, 3) (D) (0, 1)

10. (1, 3) is in the solution region of: (3 times)
(A) $x + y > 0$ (B) $x + y < 0$ (C) $x + y = 2$ (D) $x - y = 0$

11. The inequality $2x + 3y < 5$ is satisfied by point: (5 times)
(A) (1, 1) (B) (-2, 1) (C) (1, 2) (D) (-2, 3)

12. $x = 0$ is in the solution of the inequality (3 times)
(A) $2x + 1 > 0$ (B) $2x + 1 < 0$ (C) $2x + 1 \leq 0$ (D) $2x - 1 > 0$

13. $x = 5$ is not in the solution of: (5 times)
(A) $x + 4 > 0$ (B) $2x + 3 < 0$ (C) $x - 4 > 0$ (D) $x > 0$

14. $x = 5$ is the solution of inequality.
(a) $2x - 3 > 0$ (b) $2x + 3 < 0$ (c) $x + 4 < 0$ (d) $x < x$

15. Point (1, 2) lies in the solution region of the inequality.
(a) $2x + y > 5$ (b) $x + 3y > 5$ (c) $2x + y < 3$ (d) $2x + y > 6$

16. $x = -3$ is the solution of the inequality.
(a) $2x - 1 > 0$ (b) $2x + 1 > 0$ (c) $x + 4 < 0$ (d) $2x - 1 < 0$

17. Solution set of inequality $2x < 3$ is.
(a) $(-\infty, \frac{3}{2})$ (b) $(\frac{3}{2}, \infty)$ (c) $(-\infty, \infty)$ (d) $(-3/2, 3/2)$

18. Solution of inequality $x + 2y < 6$ is.
(a) (1, 3) (b) (1, 4) (c) (1, 5) (d) (1, 1)

19. Which one of the following points satisfies $x + 2y < 6$
(a) (4, 1) (b) (3, 1) (c) (1, 3) (d) (1, 4)

20. The solution set of $x < 4$ =
(a) $0 < x < 4$ (b) $10 < x < 5$ (c) $-\infty < x < 4$ (d) $4 < x < \infty$

21. Solution of $x < \frac{-3}{2}$ is:
(A) $\left(-\infty, \frac{-3}{2}\right)$ (B) $\left(\frac{-3}{2}, \infty\right)$ (C) $\left(\frac{-3}{2}, \frac{3}{2}\right)$ (D) $(-\infty, \infty)$

22. $2x + 3y < 0$ is:
(A) An equation (B) Inequality (C) Identity (D) Not identity

23. The point (-1,2) satisfies the inequality = : **(2 times)**

(A) $x - y > 4$ (B) $x - y \geq 4$ (C) $x + y < 4$ (D) $x + y > 4$

24. The point (1,3) lies in the solution region of the inequality:

(A) $x + y < 2$ (B) $x + y < 0$ (C) $x - y < 2$ (D) $x - y > 0$

25. Solution set of inequality $2x - 3 \geq 0$ equals

(A) $\left[\frac{3}{2}, \infty\right)$ (B) $\left[\frac{3}{2}, \infty\right]$ (C) $\left[\frac{2}{3}, \infty\right[$ (D) $\left[\frac{2}{3}, \infty\right]$

26. The associated equation of inequality $x + 2y < 6$ is:

(A) $x + 2y = 6$ (B) $x - 2y = 6$ (C) $x + 2y = -6$ (D) $x - 2y = -6$

27. $x + 2y > 6$ is not satisfied by:-

(A) (2,3) (B) (2,2) (C) (3,2) (D) (3,3)

28. A function which is to be maximized or minimized is called: **(4 Times)**

(A) subjective function (B) qualitative function

(C) objective function (D) quantitative function

29. (1 , 0) is the solution of inequality

(A) $7x + 2y < 8$ (B) $x - 3y < 0$ (C) $3x + 5y < 6$ (D) $-3x + 5y > 2$

30. $x = 0$ is not in the solution of inequality

(A) $2x + 3 > 0$ (B) $x + 4 > 0$ (C) $x + 5 > 0$ (D) $2x + 3 < 0$

31- Point (1, 2), satisfies the inequality:

(A) $2x + y > 5$ (B) $2x + y \geq 5$ (C) $2x + y < 3$ (D) $2x + y < 5$

32- The graph of $2x \geq 3$ lies in:

(A) Upper Half Plane (B) Lower Half Plane (C) Left Half Plane (D) Right Half Plane

33- $2x - 8 \leq 0$ is:

(A) Equation (B) identity (C) inequality (D) curve

34- The feasible solution which maximizes or minimizes the objective function is called:

(A) Exact solution (B) Optimal solution (C) Final solution (D) Objective solution

35- (1, 2) is one of the solution of inequality:

(A) $2x + y > 5$ (B) $2x - y \geq 5$ (C) $2x + y < 3$ (D) $2x + y < 5$

36. To find optimal solution we evaluate the objective function at:

(a) One point (b) Origin (c) Some points (d) Corner Points

37. $(1,1)$ is solution of :

(a) $x+y<1$ (b) $2x+y<1$ (c) $2x-y<1$ (d) $x-y<1$

38. The non-negative constrains are called

(a) Decision Variables (b) Feasible Solution set (c) Optimal Solution (d) Associated Equation

ANSWERS TO THE MULTIPLE CHIOCE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13
D	C	B	A	B	A	D	A	A	A	B	A	B
14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	D	A	D	B	C	A	B	C	C	A	A
27	28	29	30	31	32	33	34	35	36	37	38	
B	C	A	D	D	D	C	B	D	D	D	A	

SHORT QUESTIONS OF CHAPTER-5 ACCORDING TO ALP SMART SYLLABUS-2020

1. **Define feasible solution set.** **(4 times)**

Sol:- **Feasible solution set:-** The region restricted to Ist quadrent is called feasible region and a set consisting of all feasible solutions of the system of linear inequalities is called feasible solution set.

2. **Draw the graph of $3x+2y\geq 6$.** **(4 times)**

Sol:- $3x+2y\geq 6$.......(i)

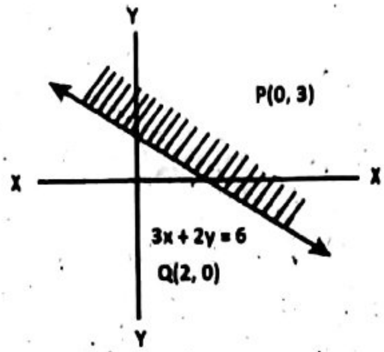
The Associated Equation of (i) is 3x+2y=6.............(ii)

Put x= 0 in (ii) $\Rightarrow 2y=6$ as $y=3$ and point is p(0,3)

Put y= 0 $\Rightarrow 3x=6$ as $x=2$ and second point is Q(2,0) is obtained.

Put O(0,0) as test point in (i).

$0+0\geq 6$ False.



3. **Define objective function and optimal solution.** **(19 times)**

Sol:- **Objective function:-**

A function which is to be maximize or minimize is called objective function.

Optimal solution:-

The feasible solution which maximize and minimize the objective function is called optimal solution.

4. **Graph the solution set of $5x-4y\leq 20$.** **(8 times)**

Sol:- $5x-4y\leq 20$....(i)

The Associated Equation of (i) is $5x-4y=20$.........(ii)

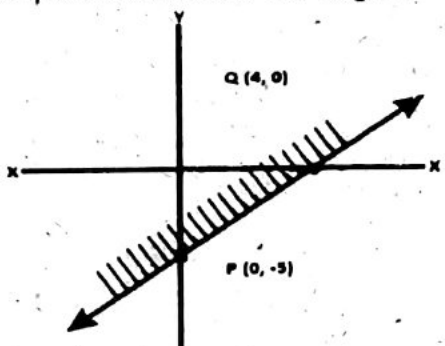
Put x= 0 in (i) $\Rightarrow -4y=20$ or $y=-5$ and point P(0,-5)

Put y= 0 in (ii) $\Rightarrow 5x=20$ or $x=4$ and second point Q(4,0) is obtained.

Put O(0,0) as test point in (i)

$0-0\leq 20$ true

So solution is shaded partion towards the origin.



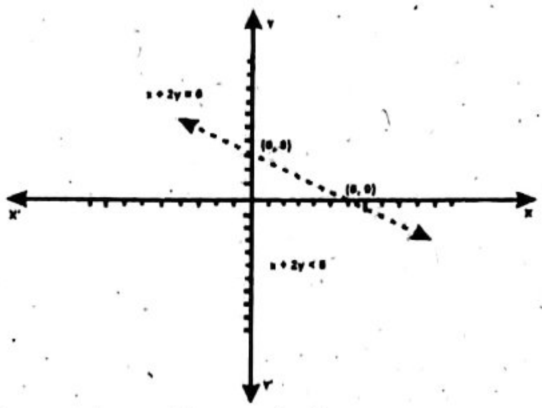
5. **Graph the inequality $x+2y<6$.** **(3 times)**

Sol:- The associated equation of the inequality

$x+2y<6$..........(i)

Or $x+2y=6$..........(ii)

The line (ii) intersects the x-axis and y-axis at (6,0) and (0,3) respectively. As no point of the line (ii) is a solution of the inequality (i), so the graph of the line (ii) is shown by dashes. We take O(0,0) as a test point because it is not on the line (ii).



6. **Graph the solution region of $2x + y \geq 2$**

Sol: $2x + y \geq 2$ (i)

The A.E of (i) is $2x + y = 2$ (ii)

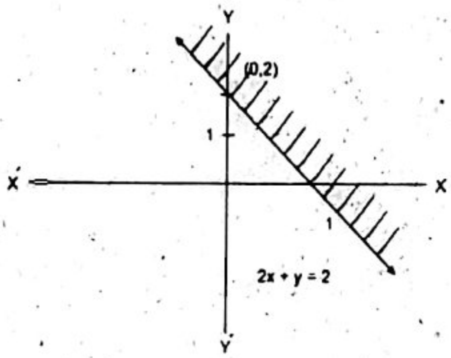
Put $x = 0$ in (ii) $\Rightarrow y = 2$ and point (0,2)

Put $y = 0$ in (ii) $\Rightarrow 2x = 2$ as $x = 1$ and point (1 , 0) is obtained

Put 0 (0 , 0) as test point in (i)

$0 + 0 \geq 2$ false

Graph



7. **Graph the solution set of inequality $2x + y \leq 6$** **(3 times)**

Sol: $2x + y \leq 6$ (i)

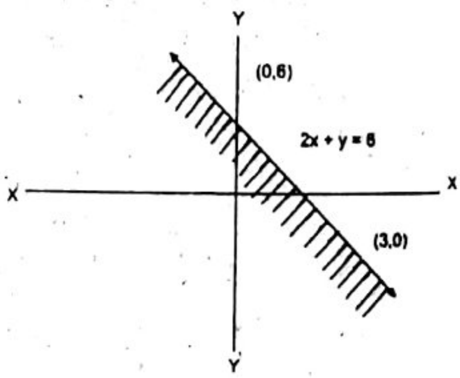
The Associated Equation of (i) is $2x + y = 6$ $\Rightarrow$ (ii)

Put $x = 0$ is (ii) $y = 6$ and point (0 , 6)

Put $y = 0$ in (ii) $2x = 6$ as $x = 3$ and point (3 , 0) is obtained

Put 0 (0,0) as test point in (i)

$0 + 0 < 6$ True



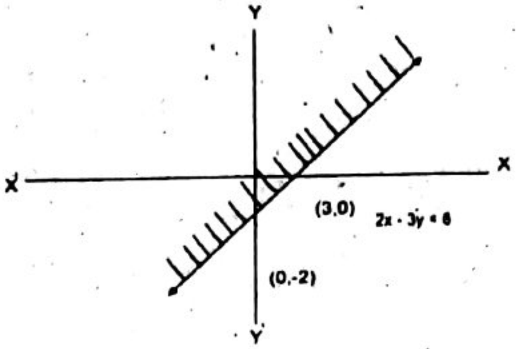
8. **Define vertex of the solution region.**

Sol: **Vertex of solution region** A point of a solution region where two of its boundary line interest is called vertex of solution region.

9. **Graph the region of following inequality $2x - 3y \leq 6$.**

Sol: $2x - 3y \leq 6$ (i)

The A.E of (i) is $2x - 3y = 6$ (ii)
Put $x = 0$ in (ii) $\Rightarrow -3y = 6$ as $y = -2$ and point $(0, -2)$
Put $y = 0$ in (ii) $\Rightarrow 2x = 6$ as $x = 3$ and point $(3, 0)$ is obtained
Put $O(0, 0)$ as test point in (i)
$0 - 0 < 6$ True

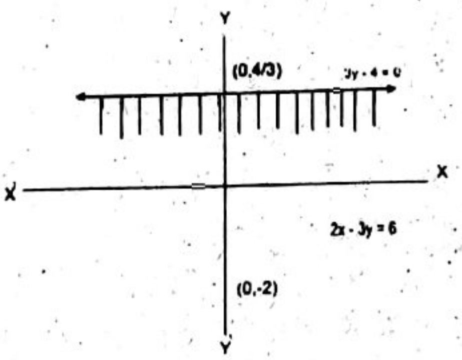


10. **Graph the solution set $3y - 4 \le 0$ in xy – plane.** **(2 times)**

Sol: Given inequality is

$3y - 4 \le 0$ in xy plane
The A.E of (i) is $3y - 4 = 0$
$3y = 4$
$Y = 4/3 \Rightarrow (0, \frac{4}{3})$

Put $O(0, 0)$ as test point in (i)
$0 - 4 \le 0$
$-4 < 0$ True

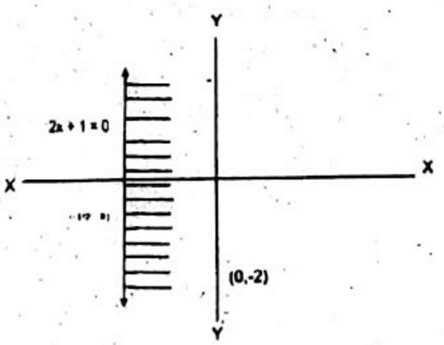


11. **Graph the solution region of $2x + 1 \ge 0$** **(2 times)**

Sol:

Given $2x + 1 \ge 0$ (i)
The A.E of (i) is $2x + 1 = 0$
$2x = -1$
$x = -1/2$
Point $(-1/2, 0)$

Put $O(0, 0)$ as test point in (i)
$0 + 1 \ge 0$ True



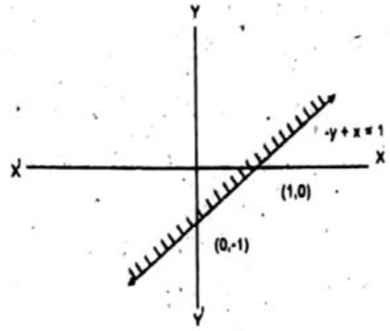
12. **What are problem constraints?** **(3 times)**

Sol: **Problem Constraints:** In a certain problem from everyday life each linear inequality concerning the problem is called problem constraints.

13. **Shade the solution region of inequality $-y + x \le 1$**

Sol: Given $-y + x \le 1$ (i)
The A.E of (i) is $-y + x = 1$ (ii)
Put $x = 0$ in (ii) $-y = 1$ as $y = -1$ and point $(0, -1)$
Put $y = 0$ in (ii) $x = 1$ and point $(1, 0)$ is obtained

Put O (0 , 0) as test point in (i)
$-0 + 0 \leq 1$ true



14. Graph the solution region of linear inequality $x + y \leq 5$.

Sol: **Given**

$x + y \leq 5$ ______________ (1)

Associative equation

$x + y = 5$ ______________ (2)

X-Intercept:

Put y = 0, in eq (2)
$x + 0 = 5$
$x = 5$
$P_1(x , y) = (5, 0)$

Y-Intercept:

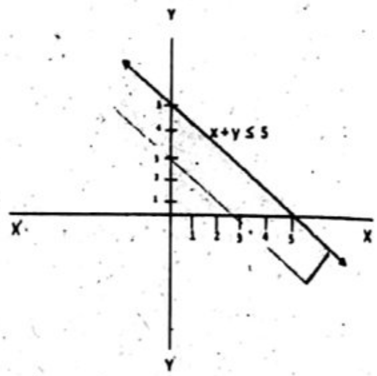
Put x = 0 , in eq (2)
$0 + y = 5$
$Y = 5$
$P_2(x , y) = (0 , 5)$

Testing:

Put x = 0 , y = 0 , in eq (1)
$0 + 0 < 5$
$0 < 5$ true

Since statement (1) is satisfy.
So shade is near the origin.

Graph



15. Graph the solution region of linear inequality $x + y \geq 5$.

Sol **Given**

$x + y \geq 5$ ______________ (1)

Associative equation

$x + y = 5$ ______________ (2)

X-Intercept:

Put y = 0, in eq (2)
$x + 0 = 5$
$x = 5$
$P_1(x , y) = (5, 0)$

Y-Intercept:

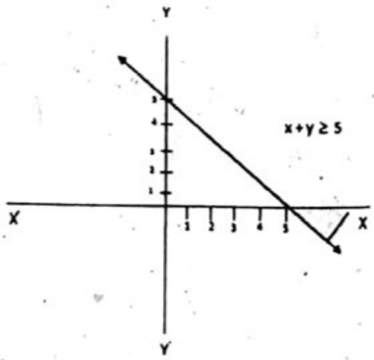
Put x = 0 , in eq (2)
$0 + y = 5$
$Y = 5$
$P_2(x , y) = (0 , 5)$

Testing:

Put x = 0 , y = 0 , in eq (1)
$0 + 0 > 5$
$0 > 5$ true

Since statement (1) does not satisfy. So shade is for the origin.

Graph



16. Decision Variables.

Sol: The variable used in the system of linear inequality to problems of every day life are called decision variables.

OR The non-negative constrained used in a system of linear inequalities are called decision variable.

2nd Year

17. **What is an Inequality?**
Sol: An expression involving any one of the symbols $>, <, \geq, \leq$ is called inequality.

LONG QUESTIONS OF CHAPTER-5 ACCORDING TO ALP SMART SYLLABUS-2020

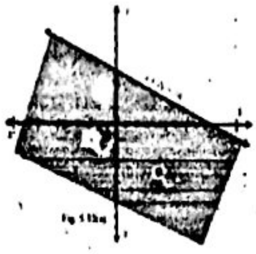
1. Maximize f(x, y) = x + 3y subject to the constraints $2x+5y\leq 30$; $5x+4y\leq 20$; $x\geq 0$; $y\geq 0$. **(C.W) (2 times)**
2. Graph the feasible region and find the corner points for the following system of inequalities. $x-y\leq 3, x+2y\leq 6, x\geq 0, y\geq 0$ **(C.W)**
3. Maximize f(x, y) = 2x + 3y subject to the constraints $3x+4y\leq 12$; $x+2y\leq 14$; $4x-y\leq 4$; $x\geq 0$; $y\geq 0$. **(C.W)**
4. Maximize f(x, y) = 2x + 5y subject to the constraints $2y-x\leq 8$; $x-y\leq 4$; $x\geq 0$; $y\geq 0$. **(H.W) (2 times)**
5. Graph the feasible region subject to the following constraints. **(3 times)** $2x-3y\leq 6; 2x+y\geq 2; x\geq 0, y\geq 0$ **(H.W)**
6. Maximize f(x, y) = x + 3y subject to the constraints $3x+5y\geq 15$; $x+3y\geq 9$; $x\geq 0$; $y\geq 0$. **(C.W) (2 times)**
7. Graph the feasible region subject to the following constraints **(2 times)** $2x+y\leq 10; x+4y\leq 12; x+2y\leq 10; x\geq 0, y\geq 0$ **(H.W)**
8. Graph feasible region by $2x-3y\leq 6$, $2x+y\geq 2$, $x+2y\leq 8$, $x\geq 0$, $y\geq 0$ **(H.W)**
9. Graph the feasible region subject to the following constraint **(C.W) (2 times)**
   $2x-3y\leq 6$
   $2x+3y\leq 12$
   $x\geq 0, y\geq 0$
10. Maximize f(x,y) = 2x + 5y subject to the constraints. **(H.W) (3 times)**
    $2y-x\leq 8$; $x-y\leq 4$
    $X\geq 0$; $y\geq 0$
11. Graph the feasible region of the following system of linear inequalities. Also find Corner Points $2x-3y\leq 6$, $2x+3y\leq 12$, $x\geq 0$, $y\geq 0$ **(C.W)**
12. Minimize $f(x, y) = 3x+y$; subject to constraints $3x+5y\geq 15$; $x+3y\geq 9$; $x\geq 0$; $y\geq 0$ **(C.W)**
13. Graph the feasible region and find the corner points for the following system of inequalities subject to constraint: $x-y\leq 3$; $x+2y\leq 6$, $x\geq 0$, $y\geq 0$ **(C.W)**
14. Maximize f(x , y) = x + 3y subject to the constraints **(H.W)**
    $2x+5y\leq 30$, $5x+4y\leq 20$, $x\geq 0$, $y\geq 0$
15. Graph feasible region of linear inequalities $2x+y\leq 10$, $x+4y\leq 12$, $x+2y\leq 10$ **(C.W)**
16. Graph the feasible region and also find the corner points. **(H.W) (4 times)**
    $2x-3y\leq 6$, $2x+3y\leq 12$, $x\geq 0$, $y\geq 0$

Chapter-5 (Examples According to ALP Smart Syllabus)

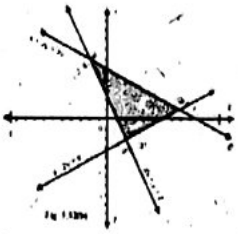
Example 2: Graph the solution region for the following system of the inequalities.

$$x-2y\leq 6,\quad 2x+y\geq 2,\quad x+2y\leq 10$$

Sol: The graph of the inequalities $x-2y\leq 6$ and $2x+y\geq 2$ have already drawn in figure 5.31(a) and 5.31(b) and their intersection is partially shown as a shaded region in figure 5.31(c) of the example 1 (Art 5.3). Following the procedure of the example 1 of Art (5.3) the graph of the inequality $x+2y\leq 10$ is shown partially in the figure 5.32(a)

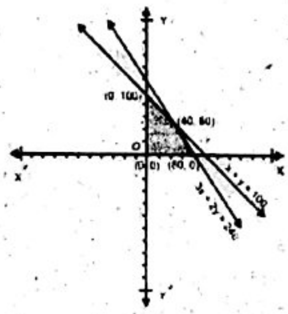


The intersection of three graphs is the required solution region graphs is the required solution region which is the shaded triangular region PQR (including its sides) shown partially in the figure 5.32(b)



Now we define a corner point of a solution region.

Example 1: A farmer possesses 100 kanals of land and wants to grow corn and wheat. Cultivation of corn requires 3 hours per kanal while cultivaton of wheat requires 2 hours per kanal. Working hours cannot exceed 240. If he gets a profit of Rs.20 per kanal for corn and Rs.15/-per kanal of wheat, how many kanals of each he should cultivate to maximize his profit?



Sol: Suppose that he cultivates x kanals of corn and y kanals of wheat. Then constraints can be written as:

$x+y\leq 100 \qquad 3x+2y\leq 240$

Non-negative constraints are $x\geq 0, y\geq 0$

Let P(x,y) be the profit function, then

P(x,y) = 20x + 15y

Now the problem is to maximize the profit function P under the given constraints.

Graphing the inequalities, we obtain the feasible region which is shaded in the figure 5.71 solving the equations x + y = 100 and 3x + 2y = 240 gives x = 240 – 2(x+y) = 240 – 200 = 40 and y = 100 – 40 = 60, that is; their point of intersection is (40,60). The corner points of the feasible region are (0,0), (0,100), (40,60) and (80,0). Now we find the values of P at the corner points.

Corner point	P(x,y) = 20x + 15y
(0,0)	P(0,0) = 20 x 0 + 15 x 0 = 0
(0,100)	P(0,100) = 20 x 0 + 15 x 100 = 1500
(40,60)	P(0,0) = 20 x 40 + 15 x6 0 = 1700
(80,0)	P(0,0) = 20 x 80 + 15 x 0 = 1600

From the above table it follows that the maximum profit is Rs. 1700 at the corner point (40,60). Thus the farmer will get the maximum profit if he cultivates 40 kanals of corn and 60 kanals of wheat.

OBJECTIVES (MCQ'S) OF CHAPTER-6 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Equation of Circle:

1. **If a plane passes through the vertex of the cone being called:**
(A) Parabola (B) Hyperbola (C) Unit circle (D) Point circle

2. **The length of the diameter of the circle $x^2+y^2=a^2$ is:** **(3 times)**
(A) a (B) 2a (C) 1 (D) 2

3. **Center of circle $4x^2+4y^2-8x+16y-25=0$:** **(2 times)**
(A) $\left(1,\frac{-3}{2}\right)$ (B) $\left(\frac{-3}{2},1\right)$ (C) (1, -2) (D) (2, 1)

4. **Radius of circle $4x^2+4y^2+8x+8y-68=0$ is:** **(3 times)**
(A) $4\sqrt{5}$ (B) $\sqrt{19}$ (C) 5 (D) 12

5. **Centre of circle $x^2+y^2+7x-3y=0$** **(2 times)**
(A) $\left(\frac{-7}{2},\frac{3}{2}\right)$ (B) $\left(\frac{7}{2},\frac{-3}{2}\right)$ (C) (7, -3) (D) (-7, 3)

6. **Center of circle $5x^2+5y^2+14x+12y-10=0$ is:** **(4 times)**
(A) $\left(-\frac{7}{5},-\frac{6}{5}\right)$ (B) $\left(\frac{7}{6},\frac{6}{5}\right)$ (C) (7, 6) (D) (-7, -6)

7. **The centre of the circle $x^2+y^2+2gx+2fy+c=0$ is:** **(3 times)**
(A) (g, f) (B) (f, g) (C) (-f, -g) (D) (-g, -f)

8. **The centre of the circle $x^2+y^2-6x+4y+13=0$ is:** **(2 times)**
(A) (-6, 4) (B) (6, -4) (C) (3, -2) (D) (-3, 2)

9. **$x^2+y^2+2gx+2fy+c=0$ is the equation of :** **(5 times)**
(A) Line (B) Ellipse (C) Hyperbola (D) Circle

10. **________ is equation of point circle:** **(2 times)**
(A) $x^2-y^2=7$ (B) $x^2+y^2=4$ (C) $x^2+y^2=0$ (D) $x^2+y^2=1$

11. **A circle is called a point circle if:**
(A) r = 1 (B) r = 0 (C) r = 2 (D) r = 3

12. **The centre of the circle $(x-1)^2+(y+3)^2=3$ is equal to.**
(a) (-1, -3) (b) (-1, 3) (c) (1, -3) (d) (1, 3)

13. **Equation of circle with centre at origin and radius of $\sqrt{5}$ is.**
(a) $x^2+y^2=\sqrt{5}$ (b) $x^2+y^2=5$ (c) $x^2+y^2=25$ (d) $x^2+(y-1)^2=5$

14. **If (-5, 3) is the centre of circle and point (7, -2) lies on it, then radius of circle will be equal to.**
(a) 3 (b) 7 (c) 5 (d) 13

15. **The circle $x^2+y^2+2gx+2fy+c=0$ passing through origin if:**
(A) c = 0 (B) c = 1 (C) c = -1 (D) c = f + g

16. **The set of all the points in a plane which are equidistant from a fixed point and fixed line is called:**
(A) Circle (B) Ellipse (C) Parabola (D) Hyperbola

17. **If the ends of the diameter of the circle are (0 , 1) and (2 , 3) then its area is:**
(A) π (B) 2π (C) 4π (D) 8π

18. **Centre of the circle having equation $x^2 + y^2 + 12x - 10y = 0$**
(A) (6 , -5)1 (B) (-6 , 5) (C) (-6 , -5) (D) (6 , 5)

19. **The co-ordinates of centre of circle $x^2 + y^2 - 6x + 4y + 13 = 0$ is equal to:**
(A) (-3 , 2) (B) (3 , -2) (C) (3 , 2) (D) (-3 , 2)

20. **If $x^2 + y^2 + 2gx + 2fy + c = 0$ represent equation of circle then radius r = (2 times)**
(A) $\sqrt{g^2 + f^2 + c}$ (B) $\sqrt{g^2 - f^2 - c}$ (C) $\sqrt{g^2 - f^2 + c}$ (D) $\sqrt{g^2 + f^2 - c}$

21. **Equation of circle with centre at origin and radius $\sqrt{5}$ is:**
(A) $x^2 + y^2 = \sqrt{5}$ (B) $x^2 + y^2 = 5$ (C) $x^2 + y^2 = 25$ (D) $(x - 3)^2 + y^2 = 5$

Topic II: Tangent and Normal Lines:

22. **Condition that the line y = mx + c is tangent to the circle $x^2 + y^2 = a^2$ is:**
(A) $c = \pm m\sqrt{1 + a^2}$ (B) $c = \pm a\sqrt{1 + m^2}$ (C) $c = \pm a\sqrt{1 - m^2}$ (D) $c = \pm m\sqrt{1 - a^2}$

Topic III: Parabola:

23. **Any chord passing through the focus of the parabola is called the :**
(A) Vertex of the parabola (B) Axis of the parabola
(C) Latus rectum of the parabola (D) Focal chord of the parabola

24. **Parabola $y^2 = 4ax, a > 0$ opens:** **(5 times)**
(A) Upward (B) Downward (C) Right side (D) Left side

25. **Focus of parabola $x^2 = -16y$** **(4 times)**
(A) (0, 4) (B) (0, -4) (C) (4, 0) (D) (-4, 0)

26. **The directrix of parabola $x^2 = -16y$ is:** **(4 times)**
(A) y − 1 = 0 (B) y + 1 = 0 (C) y − 4 = 0 (D) y + 4 = 0

27. **Focus of the parabola $x^2 = 5y$ is:** **(4 times)**
(A) $\left(0, -\frac{5}{4}\right)$ (B) $\left(0, \frac{5}{4}\right)$ (C) $\left(\frac{5}{4}, 0\right)$ (D) $\left(-\frac{5}{4}, 0\right)$

28. **The vertex of the parabola $x^2 = 4ay$ is:** **(4 times)**
(A) (a, 0) (B) (-a, 0) (C) (0, 0) (D) (0, -a)

29. **The focus of the parabola $y^2 = 4ax$ is:** **(4 times)**
(A) (a, 0) (B) (-a, 0) (C) (0, 0) (D) (0, -a)

30. **The length of the latus rectum of parabola $y^2 = 8x$ is:** **(3 times)**
(A) 2 (B) 4 (C) 6 (D) 8

31. **The vertex of parabola $y^2 = -8(x - 3)$ is:** **(3 times)**
(A) (3, 0) (B) (2, 1) (C) (3, 1) (D) (1, 0)

32. **Focus of the parabola $x^2 = 4ay$ is.**
(a) (a, 0) (b) (-a, o) (c) (o, a) (d) (0,-a)

33. **Focus of the parabola $y^2 = -4ax$ is. (2 Times)**
(a) (a, 0) (b) (-a, 0) (c) (0, a) (d) (0, - a)

34. **The line y = mx + c, will be tangent to the parabola $y^2 = 4ax$ if:**
(a) $c = -am^2$ (b) $c = \frac{a}{m}$ (c) $c = a(1 + m^2)$ (d) $c = \frac{m}{a}$

35. **The vertex of the parabola $(x - 1)^2 = 8(y + 2)$ is.**
(a) (1, -2) (b) (0, 2) (c) (2, 0) (d) (0, 0)

36. **The directrix of the parabola $x^2 = -8y$ is:**
(A) x + 2 = 0 (B) x − 2 = 0 (C) y + 2 = 0 (D) y − 2 = 0

37. **$x = at^2$, y = 2at are the parametric equations of:** **(2 times)**
(A) Ellipse (B) Circle (C) Parabola (D) Hyperbola

38. Parabola having Equation $x^2 = 4ay$ = opens:
(A) Towards left (B) Towards right (C) Upwards (D) Downwards

39. The parabola $y^2 = 4ax$, $a > 0$ opens:
(A) right (B) left (C) upward (D) downward

Topic V: Hyperbola:

40. Foci of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are: **(3 times)**
(A) $(\pm c, 0)$ (B) $(0, \pm c)$ (C) $(0, \pm ae)$ (D) $(\pm ae, 0)$

41. Asymptotes are very useful in graphing
(A) Circle (B) Parabola (C) Ellipse (D) Hyperbola

42. $x = a\sec\theta$, $y = b\tan\theta$ represents the parametric equations of:
(A) Circle (B) Parabola (C) Ellipse (D) Hyperbola

43. The co-ordinates of vertices of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ equals:
(A) $(0, \pm b)$ (B) $(\pm b, 0)$ (C) $(0, \pm a)$ (D) $(\pm a, 0)$

44- The centre of the circle $(x+3)^2+(y-2)^2 = 16$, equals:
(A) (3, -2) (B) (-3,-2) (C) (3, 2) (D) (-3, -2)

45- The eccentricity of $\frac{y^2}{4}-x^2=1$, equals:
(A) $\frac{2}{\sqrt{5}}$ (B) $\frac{-2}{\sqrt{5}}$ (C) $\frac{\sqrt{5}}{2}$ (D) $\frac{-\sqrt{5}}{2}$

46- Length of the diameter of the circle $(x+8)^2+(y-5)^2 = 80$ is:
(A) 160 (B) $4\sqrt{5}$ (C) $8\sqrt{5}$ (D) 40

47- Directrix of Parabola $x^2 = 16y$ is: **(2 times)**
(A) $x + 4 = 0$ (B) $x - 4$ (C) $y - 4 = 0$ (D) $y + 4 = 0$

48- The radius of circle $(x-5)^2+(y-3)^2 = 8$ is:
(A) 64 (B) 4 (C) $2\sqrt{2}$ (D) 2

49- The line $y = mx + c$ is tangent to the parabola $y^2 = 4ax$ if c = ?:
(A) $\frac{m}{a}$ (B) $\frac{-b}{a}$ (C) $\frac{a}{m}$ (D) $\frac{1}{ma}$

50- The foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are:
(A) $(\pm a, 0)$ (B) $(0, \pm a)$ (C) $(0, \pm ae)$ (D) $(\pm ae, 0)$

51- The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle with centre:
(A) (-g, -f) (B) (-f, +g) (C) (f, g) (D) (0, 0)

52- Axis of the parabola $x^2 = 4ay$ is:
(A) $y = 0$ (B) $x = 0$ (C) $x = y$ (D) $x = 1$

53- The straight line $y = mx + c$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if:
(A) $c^2 = a^2m^2 - b^2$ (B) $c^2 = b^2m^2 + a^2$ (C) $c^2 = b^2m^2 - a^2$ (D) $c^2 = a^2m^2 + b^2$

54- Vertices of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $a > b$ are:
(A) $(\pm a, 0)$ (B) $(0, \pm a)$ (C) $(0, \pm b)$ (D) $(\pm b, 0)$

55- Vertex of the parabola $y^2 = 4ax$ is:

(A) $(0, 0)$ (B) $(a, 0)$ (C) $(0, a)$ (D) (a, a)

56. If P(7, -2) lies on circle with centre (-5, 3), then its radius is:

(a) 13 (b) $\sqrt{13}$ (c) 17 (d) $\sqrt{17}$

57. If a = b then equation $\frac{x^2}{b^2}+\frac{y^2}{b^2}=1$ represents.

(a) Ellipse (b) Parabola (c) Hyperbola (d) Circle

58. Length of Latus Rectum of Parabola $x^2 = 5y$ is:

(a) 5 (b) 20 (c) $\frac{5}{4}$ (d) 10

59. For hyperbola value of eccentricity e is:

(a) 1 (b) Less than 1 (c) Greater than 1 (d) 0

60. Ecentricity e of circle is:

(a) e<1 (b) e=1 (c) e>1 (d) e=0

61. The radius of circle $x^2+y^2=5$

(a) 25 (b) $\sqrt{5}$ (c) 5 (d) (0,0)

62. The vertex of the parabola y^2+16x is:

(a) $(0,0)$ (b) $(1,0)$ (c) $(0,1)$ (d) $(1,1)$

63. Two circles are said to be concentric circles if they have:

(a) Same radius (b) Different center (c) Same center (d) Same diameter

64. Directrix of parabola $x^2=20y$ is:

(a) $x=10$ (b) $x=5$ (c) $y=-5$ (d) $x=-5$

65. The length of diameter of the circle $x^2+y^2-4x-12=0$ is :

(a) 6 (b) 7 (c) 8 (d) 9

66. Slope of tangent to parabola $y^2=4ax$ at $(a,2a)$ is:

(a) 3 (b) 2 (c) -1 (d) 1

67. If the line $6x+4y+c=0$ passes through the centre of circle $x^2+y^2+2x+3=0$, then value of 'c' will be

(a) -6 (b) 6 (c) -4 (d) 4

68. $x.y=1$ represents

(a) Circle (b) Parabola (c) Ellipse (d) Hyperbola

69. The length of tangent from $(0,1)$ to the circle $x^2+y^2+6x-3y+3=0$ is

(a) 2 (b) 3 (c) 4 (d) 1

ANSWERS TO THE MULTIPLE CHIOCE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14
d	b	c	b	a	a	d	c	d	c	b	c	b	d
15	16	17	18	19	20	21	22	23	24	25	26	27	28
a	c	b	b	b	d	b	b	d	c	b	c	b	c
29	30	31	32	33	34	35	36	37	38	39	40	41	42
a	a	A	c	b	b	a	d	c	c	a	a	d	d

43	44	45	46	47	48	49	50	51	52	53	54	55	56
d	b	c	c	c	c	C	d	a	b	d	a	a	a
57	58	59	60	61	62	63	64	65	66	67	68	69	
d	c	c	d	b	a	c	c	c	d	b	d	d	

SHORT QUESTIONS OF CHAPTER-6 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Equation of Circle:

1. **Show that $5x^2+5y^2+24x+36y+10=0$ represents a circle. Also find its center and radius.** (Example No. 2 pg.# 2) **(C.W)** **(2 times)**

Sol: $5x^2+5y^2+24x+36y+10=0$

$x^2+y^2+\frac{24}{5}x+\frac{36}{5}y+2=0$ This is an equation of circle in the general form.

Hence.

$g=\frac{12}{5}, f=\frac{18}{5}, c=2$

Thus cetre = $(-g,-f)=\left(\frac{-12}{5},\frac{-18}{5}\right)$

Radius r = $\sqrt{g^2+f^2-c}$

$=\sqrt{\frac{144}{25}+\frac{324}{25}-2}$

$=\sqrt{\frac{418}{25}}=\frac{\sqrt{418}}{5}$

$$x^2+\frac{24}{5}x+\left(\frac{12}{5}\right)^2+y^2+\frac{36}{5}y+\left(\frac{18}{5}\right)^2+2-\left(\frac{12}{5}\right)^2-\left(\frac{18}{5}\right)^2=0$$

$$\left(x+\frac{12}{5}\right)^2+\left(y+\frac{18}{5}\right)^2+2-\frac{144}{25}-\frac{324}{25}=0$$

$$\left(x+\frac{12}{5}\right)^2+\left(y+\frac{18}{5}\right)^2+\frac{50-144-324}{25}=0$$

$$\left(x+\frac{12}{5}\right)^2+\left(y+\frac{18}{5}\right)^2-\frac{418}{25}=0$$

Which is equation of circle with center $\left(\frac{-12}{5},\frac{-18}{5}\right)$ and radius $\frac{\sqrt{418}}{5}$

2. **Find an equation of circle with ends of a diameter at (-3 , 2) and (5 , -6). (H.W) (2 times)**

Sol: Centre of circle will be midpoint of line joining. (-3 , 2) and (5 , 6)

Centre = $\left(\frac{5-3}{2},\frac{-6+2}{2}\right)=\left(\frac{2}{2},\frac{-4}{2}\right)=(1,-2)$

Radius = $\sqrt{(1+3)^2+(-2-2)^2}$

$=\sqrt{16+16}=\sqrt{16\times 2}=4\sqrt{2}$

Hence required equation of circle is

$(x-1)^2 + (y+2)^2 = (4\sqrt{2})^2$

$x^2 - 2x + 1 + y^2 + 4y + 4 = 32$

$x^2 + y^2 - 2x + 4y + 5 - 32 = 0$

$x^2 + y^2 - 2x + 4y + 5 - 32 = 0$

$x^2 + y^2 - 2x + 4y - 27 = 0$

3. Find equation of a circle with centre $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$.

(C.W) (2 times)

Sol: Given centre is $(\sqrt{2}, -3\sqrt{3})$ and radius $= 2\sqrt{2}$.

$(x-h)^2 + (y-k)^2 = r^2$

$(x - \sqrt{2})^2 + [y - (-3\sqrt{3})]^2 = (2\sqrt{2})^2$

$x^2 - 2\sqrt{2}\,x + 2 + y^2 + 6\sqrt{3}\,y + 27 = 8$

$x^2 + y^2 - 2\sqrt{2}x + 6\sqrt{3}y + 21 = 0$

4. Find the centre and radius of the circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$

(H.W) (3 times)

Sol: Given equation of circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$

Dividing by 4 on both sides

$x^2 + y^2 - 2x + 3y - \frac{25}{4} = 0$ (i)

We know that $x^2 + y^2 + 2gx + 2fy + c = 0$ (ii)

Comparing (i) and (ii)

$2g = -2$, if $= 3$

$g = -1$ $f = 3/2$

$c = -\frac{25}{4}$

We know that

$r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{(-1)^2 + \left(\frac{3}{2}\right)^2 + \frac{25}{4}}$

$r = \sqrt{1 + \frac{9}{4} + \frac{25}{4}} = \sqrt{\frac{38}{4}} = \sqrt{\frac{19}{2}}$

5. Find centre and radius of circle $5x^2 + 14x + 12y - 10 = 0$ (C.W) (2 times)

Sol: Given equation of circle is

$5x^2 + 14x + 12y - 10 = 0$

Dividing by 5 on both sides

$x^2 + y^2 + \frac{14}{5}x + \frac{12}{5}y - 2 = 0$ (i)

We know that

$x^2 + y^2 + 2gs + 2fy + c = 0$ (ii)

Comparing (i) and (ii)

$2g = \frac{14}{5}$, $2f = \frac{12}{5}$

$g = 7/5$, $f = 6/5$, $c = -2$

Now centre $= (-g, -f) = \left(-7/5, -\frac{6}{5}\right)$

Radius $= r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{\left(-\frac{7}{5}\right)^2 + \left(-\frac{6}{5}\right)^2 + 2}$

$= \sqrt{\frac{49}{25} + \frac{36}{25} + 2} = \sqrt{\frac{49+36+50}{25}}$

$= \sqrt{\frac{135}{25}}$

$r = \sqrt{\frac{27}{5}}$

Topic II: Tangent & Normal Lines:

6. Find the length of the Tangents drawn from the point (-5 , 4) to the circle $5x^2 + 5y^2 - 10x + 15y - 131 = 0$ **(H.W) (2 times)**

Sol: Given equation of circle is: $5x^2 + 5y^2 - 10x + 15y - 131 = 0$

Dividing by 5 on both sides

$x^2 + y^2 - 2x + 3y - \frac{131}{5} = 0$

Let d be the length of tangent from point (-5 , 4) is.

$d = \sqrt{(-5)^2 + (4)^2 - 2(-5) + 3(4) - \frac{131}{5}}$

$d = \sqrt{25 + 16 + 10 + 12 - \frac{131}{5}}$

$= \sqrt{63 - \frac{131}{5}} = \sqrt{\frac{315-131}{5}}$

$d = \sqrt{\frac{184}{5}}$

7. Write equation of tangent and normal to the circle $x^2 + y^2 = 25$at (4 , 3). **(H.W)**

Sol: Given circle equation is $x^2 + y^2 = 25$ at point (4 , 3)

Equation of tangent to circle at point (x_1 , y_1)

$x_1 x + y_1 y = a^2$

Equation of tangent to given circle at point (4 , 3) is

$4x + 3y = 25$

Equation of Normal to circle at point (x_1 , y_1) is

$y_1 x - x_1 y = 0$

Equation of Normal to given circle at point (4 , 3) is

$3x - 4y = 0$

Topic III: Parabola:

8. Find the vertex and focus of parabola $x^2 = -16y$. **(C.W) (3 times)**

Sol: Given equation of parabola is: $x^2 = -16y$

Comparing with $x^2 = -4ay$

We have $4a = 16 \Rightarrow a = 4$

Hence Focus = (0 , -a) = (0 , -4)

Vertex = (0 , 0)

9. Find equation of parabola with focus (-3 , 1) , directrix x = 3. **(C.W) (3 times)**

Sol: Focus is F(-3,1) and directrix is $x - 3 = 0$

Let P(x , y) be any point of the required parabola.

Then $|PF| = |PM|$

$\Rightarrow \sqrt{(x-(-3))^2 + (y-1)^2} = \frac{|x-3|}{\sqrt{1^2+0}}$

$\sqrt{(x+3)^2 + (y-1)^2} = |x-3|$

Requiring on both sides

$(x+3)^2 + (y-1)^2 = (x-3)^2$

$x^2 + 6x + 9 + (y+1)^2 = x^2 - 6x + 9$

$(y-1)^2 = 12x$

10. Find focus and vertex of Parabola $x^2 = 4(y-1)$. **(H.W) (4 times)**

Sol: $x^2 = 4(y-1)$ (1)

Let $x = x$, $y-1 = y$, Then (1) takes the form $x^2 = 4y$, (2)

Which is also a parabola whose focus lies on $x = 0$, Now coordinates of focus of (2) are $x = 0$, $y = 1$

i.e. $x = 0$, $y - 1 = 1$

$x = 0$, $y = 2$

So coordinates of focus of parabola (1) is (0 , 2)

Now vertex of (2) has coordinates $x = 0$, $y = 0$

i.e. $x = 0$, $y - 1 = 0$

$x = 0$, $y = 1$

Hence coordinates of vertex of parabola (1) are (0,1)

Topic V: Hyperbola:

11. Find the foci of the hyperbola $\frac{x^2}{4}-\frac{y^2}{9}=1$ **(C.W)**

Sol: Given equation is $\frac{x^2}{4}-\frac{y^2}{9}=1$ .... (i)

It is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ .... (ii)

By comparing (i) and (ii)

$a^2 = 4 \Rightarrow a = 2;\quad b^2 = 9 \Rightarrow b = 3$

$\because c^2 = a^2 + b^2$

$\Rightarrow c^2 = 4+9 = 13 \qquad \Rightarrow c = \sqrt{13}$

Foci = $(\pm c, 0) = \left(\pm\sqrt{13}, 0\right)$

12. Write the standard equation of Hyperbola.

Sol **Standard equation of hyperbola is**

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

13. Find Eccentricity and Foci of the Hyperbola $\frac{y^2}{16}-\frac{x^2}{9}=1$ **(H.W).**

Sol **Given**

$$\frac{y^2}{16}-\frac{x^2}{9}=1 \text{\_\_\_\_\_\_\_\_\_\_}(1)$$

We know that

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \text{\_\_\_\_\_\_\_\_}(2)$$

From (1) and (2)

$a^2 = 16$, $b^2 = 9$

$a = \pm 4$

We know that

$c^2 = a^2 + b^2$

$c^2 = 16 + 9 = 25$

$c = \pm 5$

(i) Foci = $(0, \pm c) = (0, \pm 5)$

(ii) Eccentricity = $e = \frac{c}{a}$

$$e = \frac{5}{4} \text{ Ans.}$$

14. Find the centre and vertices of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$. (C.W)

Sol Given

$$\frac{x^2}{4} - \frac{y^2}{9} = 1 \_\_\_\_\_\_ (1)$$

We know that

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \_\_\_\_\_\_ (2)$$

From (1) and (2)

$$a^2 = 4 \quad , \quad b^2 = 9$$

$$a = \pm 2$$

(i) Centre at origin = (0,0)

(ii) Vertices = $(\pm a \ , \ 0) \ = \ (\pm 2 \ , \ 0)$

15 Find focus and vertex of parabola (H.W)

$x^2 - 4x - 8y + 4 = 0$

Sol:

$$x^2 - 4x - 8y + 4 = 0$$

$$x^2 - 4x + 4 = 8y$$

$$(x-2)^2 = 8y$$

Let: $X = x - 2$, $Y = y$

$X^2 = 4aY$

$4a = 8$

$\Rightarrow a = 2$

i) Focus: F (0, a)

$\Rightarrow X = 0$, $Y = a$

$x - 2 = 0$, $y = 2$

$x = 2$

$\Rightarrow$ F (2, 2)

Vertex: (0, 0)

$\Rightarrow X = 0$, $Y = 0$

$x - 2 = 0$, $y = 0$

$x = 2$

$\Rightarrow$ V (2, 0)

16 Find foci and vertices of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Sol: The given equation of ellipse is

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \quad \text{------------------------- (i)}$$

We know that standard equation f ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{------------------------- (ii)}$$

Comparing (i) and (ii)

$a^2 = 9$, $b^2 = 4$

$\Rightarrow$ $a = \pm 3$, $b = \pm 2$

From $c^2 = a^2 - b^2$

$c^2 = 9 - 4$

$c^{21} = 5$

$c^2 = \pm\sqrt{5}$

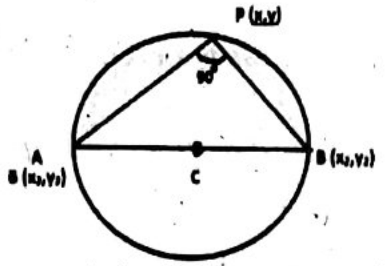
i) Foci: $F(-\sqrt{5}, 0), F'(\sqrt{5}, 0)$

II) Vertices: $A(-3,0), A'(3,0)$

17. Find an equation of the circle having the joining of $A(x_1, y_1)$ and $B(x_2, y_2)$.

(C.W)

Sol:



Since $\overline{AB}$ is a diameter of the circle so its mid point is C let $P(x_1y)$ be any point on the circle then

Slope of AP = $= \dfrac{y-y_1}{x-x_1}$ and

Slope of BP $= \dfrac{y-y_2}{x-x_2}$

Now By condition of perpendicularly

$$\frac{(y-y_1)}{(x-x_1)} \times \frac{(y-y_2)}{(x-x_2)} = -1$$

$$\Rightarrow -(x-x_1)(x-x_2) = (y-y_1)(y-y_2)$$

OR $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

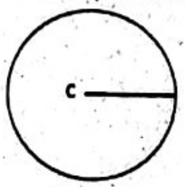
Which is required.

18. Define a circle.

Sol: The set of all points in the plane that are equidistance form a fixed point is called a circle. Fixed point is called centre of the circle.

19. What is the point circle.

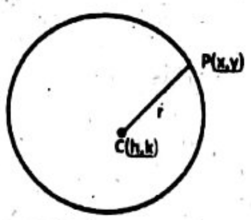
Sol: The circle with radius zero is called point circle and its equation is



$x^2 + y^2 = 0$

20. Derived standard eq. of circle.

Sol: Consider a circle with center at $c(h,k)$ and radius '"r" take any point $P(x,y)$ on the boundary of the circle then using distance formula



$|CD| = \sqrt{(x-h)^2 + (y-k)^2}$

Squaring on both sides.

$$|CD|^2 = (x-h)^2 + (y-k)^2 \quad \because |CD| = r$$

so $r^2 = (x-h)^2 + (y-k)^2$

Which is standard equation of circle.

21. Find an equation of the parabola having its focus at the origion & directrix parallel to x-axis.

Sol: $\because$ directrix is $\parallel$ to x-axis

$\therefore y = h$ (any value)

$\Rightarrow y - h = 0$

also $F(0,0)$

so required equation is $|PF| = \frac{|y-h|}{\sqrt{0^2+1^2}}$

$\Rightarrow \sqrt{(x-0)^2+(y-0)} = \frac{|y-h|}{1}$

$\Rightarrow \sqrt{x^2+y^2} = (y-h)$

taking square on both sides.

$x^2+y^2 = (y-h)^2$

$\Rightarrow x^2 + \cancel{y^2} = \cancel{y^2} + h^2 - 2hy$

$\Rightarrow x^2 = h^2 - 2hy$

$\Rightarrow x^2 + 2hy - h^2 = 0$

Which is required.

22. Define Conic.

Sol: I. If $e = 1$ then conic is a parabola.

II. If $0 < e < 1$ then the conic is an ellipse.

III. If $e > 1$ then the conic is a Hyperbola.

23. Find focus and directrix of parabola, $y^2 = 8x$

Sol: Given $y^2 = 8x$

$\because y^2 = 4ax$

$\Rightarrow 4a = 8 \Rightarrow a = 2$

So Focus : $F(a,0) = F(2,0)$

and Direction: $x = -a$

$\Rightarrow x = -2$

$\Rightarrow x + 2 = 0$

LONG QUESTIONS OF CHAPTER-6
ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Equation of Circle:

1. Find an equation of a circle passing through A(-3, 1) with radius 2 and centre at 2x – 3y + 3 = 0 **(C.W) (2 times)**
2. Find an equation of the circle passing through the points A(1, 2) , B(1, -2) and touching the line x + 2y + 5 = 0 (Example No. 6 pg . No. 454)
3. Show that the circles $x^2+y^2+2x-8=0$ and $x^2+y^2-6x+6y-46=0$ touch internally. **(4 times)**
4. Write the equation of circle passing through the given points. **(C.W)**
   A(-7, 7), B(5, -1) , C(10, 0)
5. Find the coordinates of the points of intersection of the line x + 2y = 6 with the circle $x^2+y^2-2x-2y-39=0$ **(C.W) (2 times)**
6. Show that the circles $x^2 + y^2 + 2x - 2y - 7 = 0$ and $x^2 + y^2 - 6x + 4y + 9 = 0$ touch externally. **(C.W) (3 times)**

Topic II: Tangent & Normal Lines:

7. Find equation of the tangents of the circle $x^2+y^2=2$ perpendicular to the line $3x+2y=6$. **(C.W) (3 times)**
8. Show that the line $2x+3y-13=0$ is tangent to the circle $x^2+y^2+6x-4y=0$ **(5 times)**
9. Find the length of chord cut off from the line $2x+3y=13$ by the circle $x^2+y^2=26$ **(H.W) (5 times)**
10. Find equation of the circle of radius 2 and tangent to the line $x-y-4=0$ at A (1, -3) **(H.W (3 times)**
11. Find an equation of the circle passing through the points A (1,2) and B(1, -2) and touching the line $x+2y+5=0$ **(C.W)**
12. Show that the line $3x-2y=0$ and $2x+3y-13=0$ are tangents to the circle $x^2+y^2+6x-4y=0$ **(3 times)**

Topic III: Parabola:

13. Find an equation of parabola having focus F(-3, 1) ; directrix $x=3$ **(H.W) (4 times)**
14. Find focus, vertex, and directrix of parabola $x^2-4x-8y+4=0$ **(H.W)**
15. Find an equation of the parabola whose focus is F (-3, 4) and directrix is $3x-4y+5=0$ **(C.W)**
16. Find the focus, vertex and directrix of the parabola $x+8-y^2+2y=0$ **(H.W)**
17. Write an equation of the Parabola with given elements Focus (-3, 1) and Directrix $x=3$ **(C.W)**

Chapter-6 (Examples According to ALP Smart Syllabus)

Example 3: (Page#260) Write equation of two tangents from (2,3) to the circle $x^2+y^2=9$

Sol: Any tangent to the circle is:

$$y=mx+3\sqrt{1+m^2}$$

If it passes through (2,3) then

$$3=2m+3\sqrt{1+m^2}$$

$$(3-2m)^2=9(1+m^2)$$

$$9-12m+4m^2=9+9m^2$$

$$5m^2+12m=0 \quad i.e., \quad m=0,\frac{-12}{5}$$

Inserting these values of m into (1). We have equations of the tangents from (2,3) to the circle as:

For $m=0: y=0.x+3\sqrt{1+0}$

Or $y=3$

For $m=\frac{-12}{5}: y=\frac{-12}{5}x+3\sqrt{1+\frac{144}{25}}=\frac{-12}{5}x+\frac{39}{5}$

Or $5y + 12x - 39 = 0$

Example 2: (Page#277) Find an equation of the parabola whose focus is F(-3,4) and directrix is $3x - 4y + 5 = 0$

Sol: Let P(x,y) be a point on the parabola. Length of the perpendicular $|PM|$ from P(x,y) to the directrix $3x - 4y + 5 = 0$ is

$$|PM| = \frac{|3x-4y+5|}{\sqrt{3^2+(-4)^2}}$$

By definition, $|PF| = |PM|$ or $|PF|^2 = |PM|^2$

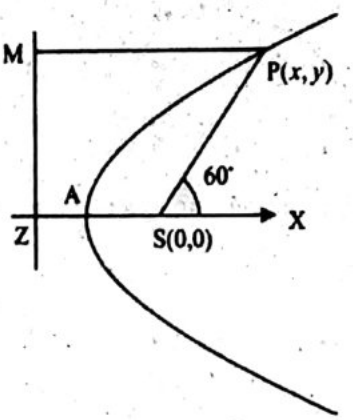
Or $(x+3)^2 + (y-4)^2 = \dfrac{(3x-4y+5)^2}{25}$

Or $25(x^2+6x+9+y^2-8y+16) = 9x^2+16y^2+25-24xy+3x-40y$

Or $16x^2+24xy+9y^2+120x-160y+600=0$

Is an equation of the required parabola.

Example 4: (Page#279) A comet has a parabola orbit with the sum at the focus. When the comet is 100 million km from the sun, the line joining the sun and the comet makes an angle of 60° with the axis of the parabola. How close will the comet get to the sun?



Sol: Let the sun S be the orgin. If the vertex A of the parabola ZM has coordinates (-a,0) then directrix. Of the parabola is x = -2a, (a>0)

If the comet is at P(x,y) then by definition $|PS| = |PM|$

i.e., $x^2 + y^2 = (x + 2a)^2$

or $y^2 = 4ax + 4a^2$ is orbit of the comet

$|PS| = \sqrt{x^2+y^2}$

$= x + 2a = 100,000,000$

The comet is closest to the sun when it is at A.

Now $x = PS\cos 60^\circ$

$$|x| = \frac{|PS|}{2} = \frac{x+2a}{2}$$

Or $\frac{x+2a}{2} = \frac{2}{1}$ or $\frac{x+2a}{2} = 2, (|x| = |-2a| = 2a)$

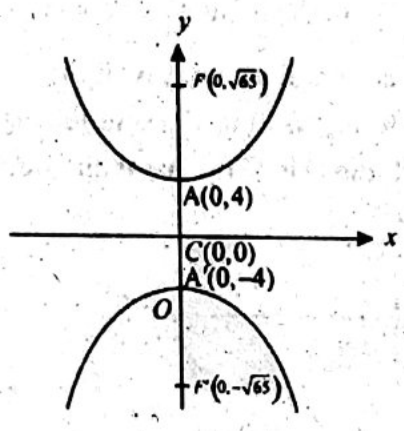
Or $\frac{100,000,000}{2a} = 2$

Or $a = 25,000,000$

Example 3: (Page#296) Find the eccentricity, the coordinates of the vertices and foci of the asymptotes of the hyperbola.

$$\frac{y^2}{16} - \frac{x^2}{49} = 1 \qquad (1)$$

Also sketch its graph.



Sol: The transverse axis of (1) lies along the y-axis. Coordinates of the vertices are $(0, \pm 4)$

Here a = 4, b = 7 so that from $c^2 = a^2 + b^2$, we get

$C^2 = 16 + 49$ or $c = \sqrt{65}$

Foci are: $(0, \pm\sqrt{65})$

Ends of the conjugate axis are $(0, \pm 7)$

Eccentricity $= \frac{c}{a} = \frac{\sqrt{65}}{4}$

$x = \pm 7$, $y = \pm 4$

The graph of the curve is as shown.

OBJECTIVES (MCQ'S) OF CHAPTER-7
ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Vector in Space:

1. The unit vector of a Vector V is : **(4 times)**

(A) $\frac{\underline{v}}{|\underline{v}|}$ (B) $\underline{v}|\underline{v}|$ (C) $\frac{|\underline{v}|}{\underline{v}}$ (D) $\frac{\underline{v}}{|\underline{v}|^2}$

2. The magnitude of vector is also called its : **(3 times)**

(A) Parameter (B) Variable (C) Point (D) Norm

3. Unit vector in the direction of $\underline{v}=2\underline{i}-\underline{j}$ is: **(3 times)**

(A) $\frac{2\underline{i}-\underline{j}}{2}$ (B) $\frac{2\underline{i}-\underline{j}}{\sqrt{2}}$ (C) $\frac{2\underline{i}-\underline{j}}{\sqrt{3}}$ (D) $\frac{2\underline{i}-\underline{j}}{\sqrt{5}}$

4. Magnitude of the vector $\underline{v}$ = [3, -4] **(3 times)**

(A) 3 (B) 4 (C) 5 (D) 6

5. If $P=(a_1,b_1)$ $Q=(a_2,b_2)$ then $\overrightarrow{PQ}$ is: **(3 times)**

(A) $(a_1+a_2)\underline{i}+(b_1+b_2)\underline{j}$ (B) $(a_1-a_2)\underline{i}+(b_1-b_2)\underline{j}$

(C) $(a_2-a_1)\underline{i}+(b_2-b_1)\underline{j}$ (D) $(a_1+b_1)\underline{i}+(a_2+b_2)\underline{j}$

6. If P = (2, 3) and Q = (6, -2) then $\overrightarrow{PQ}$=

(A) $4\underline{i}+5\underline{j}$ (B) $-4\underline{i}+5\underline{j}$ (C) $4\underline{i}-5\underline{j}$ (D) $5\underline{i}-4\underline{j}$

7. If $\vec{V}=2\vec{i}+\sqrt{5}\vec{j}+4k$, then $|\vec{V}|$ is equal to.

(a) $\sqrt{5}$ (b) 5 (c) 25 (d) 3

8. Magnitude of $2\underline{i}-3\underline{j}+\underline{k}$ is:

(A) $\sqrt{16}$ (B) $\sqrt{15}$ (C) $\sqrt{14}$ (D) $\sqrt{13}$

9. The unit vector of 2i + j is:

(A) $2\hat{i}-\hat{j}$ (B) $\frac{2\hat{i}+\hat{j}}{5}$ (C) $\frac{2\hat{i}+\hat{j}}{3}$ (D) $\frac{2\hat{i}+\hat{j}}{\sqrt{5}}$

10. A vector with magnitude 1 is called:

(A) Null vector (B) Unit vector (C) Zero vector (D) Constant

Topic II: Scalar Product of Vector:

11. If the vector $\underline{u}=2\underline{i}+4\underline{j}-7\underline{k}$ and $\underline{v}=2\underline{i}+6\underline{j}-x\underline{k}$ are perpendicular then x =

(A) -4 (B) 4 (C) 28 (D) 0

12. If $\bar{a}$ and $\bar{b}$ are oppositely directed then $\bar{a}.\bar{b}$ equals: **(2 times)**

(A) ab (B) −ab (C) $ab\sin\theta$ (D) $ab\cos\theta$

13. The magnitude of dot and cross product of two vector are 1 and 1 respectively. Then angle between vector is:

(A) 90° (B) 60° (C) 45° (D) 30°

14. $\hat{i}.\hat{k}=$ _____ **(5 times)**

(A) 5 (B) 4 (C) 2 (D) 0

15. If α, β, γ are the direction angles of a vector then $Cos^2\alpha + Cos^2\beta + Cos^2\gamma =$ (4 times)

(A) 0 (B) 1 (C) 2 (D) 3

16. Projection of $\overline{a} = \underline{i} - \underline{k}$ along $\overline{b} = \underline{j} + \underline{k}$ is: (3 times)

(A) $\frac{-1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{3}{\sqrt{2}}$ (D) $\frac{1}{2}$

17. For any two vectors $\underline{a}$ and $\underline{b}$ projection of $\underline{a}$ on $\underline{b}$ is. (2 Times)

(a) $\frac{\underline{a}.\underline{b}}{a}$ (b) $\frac{\underline{a}.\underline{b}}{|\underline{b}|}$ (c) $\frac{\underline{a}.\underline{b}}{b}$ (d) $\underline{a}.\underline{b}$

18. Two non zero vectors a and b are perpendicular if $\underline{a} . \underline{b} =$

(a) -1 (b) 1 (c) 2 (d) 0

19. If $\underline{v} = 2 \propto \underline{i} + \underline{j} - \underline{k}$, $\underline{v} = \underline{i} + \propto \underline{i} + 4\underline{k}$ are perpendicular, then $\propto =$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) 3

20. Which of triples can be direction angles of a single vector =

(a) 90°, 90°, 45° (b) 0°, 0°, 45° (c) 45°, 45°, 90° (d) 30°, 30°, 30°

21. For a vector $\underline{v} = a\,\underline{i} + b\,\underline{j} + c\,\underline{k}$, projection of $\underline{v}$ along $\underline{k}$ is:

(A) a (B) b (C) c (D) a+b+c

22. If the vectors $2\underline{i} + 4\underline{j} - 7k$ and $2\underline{i} + 6\underline{j} + x\underline{k}$ are perpendicular, then x equals

(A) 5 (B) 4 (C) -4 (D) 2

23. $\vec{i}.\vec{i} =$ ______

(A) 0 (B) 1 (C) 2 (D) 3

24. The angle between the vectors $2i + 3j + k$ and $2i - j - k$ is: (2 times)

(A) 30° (B) 45° (C) 60° (D) 90°

25. The direction cosines of x-axis are (2 times)

(A) 1,0,0 (B) 1,0,1 (C) 1,1,0 (D) 0,0,1

26. Projection of the vector $r = a\,\underline{i} + b\,\underline{j} + c\,\underline{k}$ on x – axis is.

(a) a (b) b (c) c (d) $\sqrt{a^2 + b^2 + c^2}$

Topic III: Vector Product:

27. The non-zero vectors $\underline{a}$ and $\underline{b}$ are parallel if $\underline{a} \times \underline{b} =$ (2 times)

(A) 1 (B) -1 (C) 0 (D) ab

28. Commutative law holds in : (4 times)

(A) Vector product
(B) Cross product in three vectors
(C) Inner product
(D) None of these

29. $\underline{u} \times \underline{v}$ is equal to: (6 times)

(A) $uv \sin\theta$ (B) $\underline{u} \times \underline{v}$ (C) $uv \cos\theta$ (D) $-\underline{v} \times \underline{u}$

30. The area of triangle whose adjacent sides are $3\hat{i} + 4\hat{j}$ and $12\hat{i} + 9\hat{j}$ is:

(A) $\frac{45}{2}$ (B) $\frac{55}{2}$ (C) $\frac{21}{2}$ (D) $\frac{75}{2}$

31. $\vec{j} \times \vec{k}$ is equal to.

(a) $\vec{i}$ (b) $-\vec{i}$ (c) 1 (d) $\vec{0}$

32. Angle between the vectors $\underline{i} + \underline{j}$ & $\underline{i} - \underline{j}$ is:

(A) 0 (B) π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

33. $\vec{j} \times \vec{k}$

(A) $\vec{i}$ (B) $-\vec{i}$ (C) 1 (D) 0

Topic IV: Application of Vector:

34. $\bar{u}\times(\bar{v}.\bar{w})$ is: (1 time)
(A) Scalar product (B) Vector product (C) Cross product (D) Meaningless

35. Volume of parallel pipped with $\underline{u}, \underline{v}, \underline{w}$ as its coterminous edges is:
(A) $\underline{u}.\underline{v}\times\underline{w}$ (B) $\frac{1}{3}(\underline{u}.\underline{v}\times\underline{w})$ (C) $\frac{1}{6}(\underline{u}.\underline{v}\times\underline{w})$ (D) $\frac{1}{2}(\underline{u}.\underline{v}\times\underline{w})$

36. $2\underline{i}\times 2\underline{j}.\underline{k}=$ (5 times)
(A) 2 (B) 3 (C) 4 (D) 0

37. $[\underline{k}, \underline{i}, \underline{j}]$ = ________ (4 times)
(A) 0 (B) -1 (C) -2 (D) 1

38. $\hat{j}.(\hat{k}\times\hat{i})$ is equal to: (5 times)
(A) $\hat{i}$ (B) 1 (C) -1 (D) $\hat{j}$

39. $\hat{i}.(\hat{j}\times\hat{k})$ is equal to: (5 times)
(A) 0 (B) 1 (C) -1 (D) 2

40. The work done by a force $\underline{F}$ during displacement $\underline{d}$ is equal to: (2 times)
(A) $\underline{d}\times\underline{F}$ (B) $\underline{F}\times\underline{d}$ (C) $-\underline{F}.\underline{d}$ (D) $\underline{F}.\underline{d}$

41. $2\hat{i}\times\hat{j}.\hat{k}=$
(a) 2 (b) 0 (c) 1 (d) -2

42. $2i\times(2k\times i)=$
(a) $4\underline{k}$ (b) -4 (c) 0 (d) $4\underline{i}$

43. If $\underline{d}$ is the displacement and $\underline{F}$ is the applied force, then work done by force $\underline{F}$ is equal to.
(a) $\underline{F}+\underline{D}$ (b) $\underline{F}\cdot\underline{d}$ (c) $\underline{F}-\underline{d}$ (d) $\underline{F}\times\underline{d}$

44. $(\underline{i}\cdot\underline{i})\times\underline{k}=$
(a) 0 (b) 1 (c) $\underline{i}$ (d) $\underline{k}$

45. What is the value of $[\underline{a}\ \underline{b}\ \underline{b}]$ =
(a) 1 (b) - 1 (c) 0 (d) 2

46. $\underline{j}\cdot\underline{k}\times\underline{i}=$
(A) $\underline{i}$ (B) $\underline{j}$ (C) $\underline{k}$ (D) 1

47. $(\underline{i}\times\underline{k})\times\underline{j}=$
(A) 1 (B) $\underline{j}$ (C) 0 (D) $\underline{i}$

48. $\underline{j}\cdot(\underline{k}\times\underline{i})$ is equal to:
(A) 0 (B) -1 (C) 1 (D) 2

49. Moment of force $\underline{F}$ about ($\underline{r}$) is: (2 times)
(A) $\underline{r}\times\underline{F}$ (B) $\underline{F}\times\underline{r}$ (C) $\underline{r}\cdot\underline{F}$ (D) $\underline{F}\cdot\underline{r}$

50. The moment of a force F acting at P about C is
(A) F x CP (B) CP x F (C) CP . F (D) OP x F

51. $[\vec{a}\,\vec{b}\,\vec{c}]$ is equal to: (4 times)
(A) 1 (B) 0 (C) -1 (D) 2

52- $2\hat{i}.(3\hat{j}\times\hat{k})$ is equal to:
(A) 0 (B) 2 (C) 4 (D) 6

53- **Cosθ =**

(A) $\frac{\hat{a}.\hat{b}}{|a||b|}$ (B) $|\bar{a}\times\bar{b}|$ (C) $\hat{a}.\times\hat{b}$ (D) $\frac{|\bar{a}\times\bar{b}|}{|\bar{a}|}$

54- **A unit vector perpendicular to the vectors $\underline{a}$ and $\underline{b}$ is:**

(A) $\frac{\underline{a}\times\underline{b}}{|\underline{a}||\underline{b}|}$ (B) $\frac{\underline{a}\times\underline{b}}{|\underline{a}\times\underline{b}|}$ (C) $\frac{|\underline{a}||\underline{b}|}{|\underline{a}\times\underline{b}|}$ (D) $\frac{|\underline{a}\times\underline{b}|}{|\underline{a}||\underline{b}|}$

55- $[\hat{k}\ \hat{i}\ \hat{j}]=$:

(A) 1 (B) 2 (C) -1 (D) -2

56- **The angle between the vectors** $2\hat{i}+3\hat{j}+\hat{k}$ **and** $2i+-\hat{j}-\hat{k}$ **is:** **(2 times)**

(A) 30^0 (B) 45^0 (C) 60^0 (D) 90^0

57- **If the vectors** $2\propto\hat{i}+\hat{j}-\hat{k}$ **and** $\hat{i}+\propto\hat{j}\_+4\hat{k}$ **are perpendicular to each other, then value of "∝" is:** **(2 times)**

(A) 3 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

58- **Length of the vector** $2\underline{i}-\underline{j}-2\underline{k}$ **is:**

(A) 2 (B) 4 (C) 3 (D) 5

59- **The direction cosines of y – axis are:** **(2 times)**

(A) (0, 1, 0) (B) (1, 0, 0) (C) (0, 0, 1) (D) (0, 0, 0)

60- **The non-zero vector '$\underline{a}$' and '$\underline{b}$' are parallel if $\underline{a}\times\underline{b}=$:**

(A) 0 (B) 1 (C) −1 (D) (a, b)

61- **If any two vectors of scalar triple product are equal then its value is:**

(A) 1 (B) 2 (C) −1 (D) 0

62. **If** $\bar{u}=\bar{v}$, **then** $\bar{u}.(\bar{v}\times\bar{w})=$

(a) 0 (b) 1 (c) − 1 (d) Cannot be calculated

63. **Direction cosines of z – axis are:**

(a) [1,0,0] (b) [1,1,1,] (c) [0,1,0] (d) [0,0,1]

64. **The triple scalar product of vectors, calculates the volume of:**

(a) Triangle (b) Parallelogram (c) Tetrahedron (d) Parallelepiped

65. **The position vector of any point in xy-plane is:**

(a) $x\underline{i}+y\underline{j}+z\underline{k}$ (b) $y\underline{j}+z\underline{k}$ (c) $x\underline{i}+y\underline{j}$ (d) $x\underline{i}+z\underline{k}$

66. **A force** $\vec{F}$ is applied at an angle of measure $\frac{\pi}{2}$ with the displacement vector $\vec{r}$. The work done will be

(a) $\vec{F}\times\vec{r}$ (b) $\vec{r}\times\vec{F}$ (c) Zero (d) $\vec{F}.\vec{r}$

67. **If** $\overline{OA}=\vec{a},\overline{OB}=\vec{b},$ *then* $\overline{AB}=$

(a) $\vec{a}-\vec{b}$ (b) $\vec{a}+\vec{b}$ (c) $\vec{b}-\vec{a}$ (d) $\vec{a}.\vec{b}$

68. **Angle between the vectors** $4\underline{i}+2\underline{j}-\underline{k}$ and $-\underline{i}+\underline{j}-2\underline{k}$ is

(a) 30^0 (b) 45^0 (c) 90^0 (d) 60^0

ANSWERS TO THE MULTIPLE CHIOCE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	d	d	c	c	c	b	c	d	b	a	b	c	d	b
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
a	b	d	c	c	c	b	b	d	a	a	c	c	d	c
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
c	c	a	d	a	c	d	b	b	d	a	a	b	a	c

46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
d	c	c	a	b	a	d	a	b	a	d	d	c	a	a
61	62	63	64	65	66	67	68							
d	a	d	d	c	c	c	c							

SHORT QUESTIONS OF CHAPTER-7 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Vector in Space:

1. Find direction of cosines of $\underline{V}=\underline{i}-\underline{j}-\underline{k}$ **(H.W)** **(2 times)**

Sol: Give $\underline{V}=\underline{i}-\underline{j}-\underline{k}$

So $|\underline{V}|=\sqrt{(1)^2+(-1)^2+(-1)}=\sqrt{1+1+1}=\sqrt{3}$

Therefore direction cosines of $\underline{V}$ are $\left(\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{-1}{\sqrt{3}}\right)$

2. If $\overline{AB}=\overline{CD}$, Find the coordinates of the point A when B(1, 2), C(-2, 5) and D(4, 11) are given. **(H.W)**

Sol: P.V of B = $\underline{i}+2\underline{j}$

P.V of C = $-2\underline{i}+5\underline{j}$

P.V of D = $4\underline{i}+11\underline{j}$

We suppose that (a, b) be coordinates of point A

Then P.V of A = $a\underline{i}+b\underline{j}$

Give $\overline{AB}=\overline{CD}$

i.e P.V of B – P.V of A = P.V of D – P.V of C

$\Rightarrow\quad \underline{i}+2\underline{j}-(a\underline{i}+b\underline{j})=(4\underline{i}+11\underline{j})-(-2\underline{i}+5\underline{j})$

$\Rightarrow\quad \underline{i}+2\underline{j}-a\underline{i}-b\underline{j})=4\underline{i}+11\underline{j}+2\underline{i}-5\underline{j}$

$\Rightarrow\quad (1-a)\underline{i}+(2-b)\underline{j}=6\underline{i}+6\underline{j}$

$\Rightarrow\quad$ 1 – a = 6 and 2 – b = 6 $\quad\Rightarrow$ a = - 5 and b = - 4

i.e (-5, -4) are required coordinates of A.

3. If $\vec{v}=3i-2j+2k, \vec{w}=5i-j+3k$ find $|3\vec{v}+\vec{w}|$. **(C.W)**

Sol: $3\vec{v}+\vec{w}=3(3i-2j+2k+5i-j+3k)$

$3\vec{v}+\vec{w}=9i-6j+6k+5i-j+3k$

$3\vec{v}+\vec{w}=14i-7j+9k$

Taking magnitude on both sides

$|3\vec{v}+\vec{w}|=\sqrt{(14)^2+(-7)^2+(9)^2}$

$|3\vec{v}+\vec{w}|=\sqrt{196+49+81}$

$|3\vec{v}+\vec{w}|=\sqrt{326}$

4. Find a vector from the point A to the origin where $\overline{AB}=4\underline{i}-2\underline{j}$ and B is the point $(-2,5)$. **(C.W)** **(3 times)**

Sol:

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{AB}$

$\overrightarrow{OA} = -2\hat{i} + 5\hat{j} - 4\hat{i} + 2\hat{j}$

$\overrightarrow{OA} = -6\hat{i} + 7\hat{j}$ $\quad\quad \because -\overrightarrow{OA} = \overrightarrow{AO}$

$\overrightarrow{OA} = -\overrightarrow{AO}$

$-\overrightarrow{AO} = -6\hat{i} + 7\hat{j}$

$\overrightarrow{AO} = 6\hat{i} - 7\hat{j}$

5. If o is the origin and $\overline{OP} = \overline{AB}$, find the point P when A and B are (-3 , 7) and (1 , 0) respectively? (C.W) (2 times)

Sol: Given points are A = (-3 , 7) , B = (1 , 0)

Suppose Coordinates of P be P= (x, y)

Now $\overline{op} = (x-0)\,\underline{i} + (y-0)\,\underline{j}$

$= x\underline{i} + y\underline{j}$

And $\overline{AB} = (1+3)\,\underline{i} + (0-7)\,\underline{j}$

$= 4\underline{i} - 7\underline{j}$

As given $\overline{OP} = \overline{AB}$

$x\underline{i} + 4\underline{j} = 4\underline{i} - 7\underline{j}$

By equality of vectors

x = 4 , y = -7

So P = (x , y) = (4 , -7)

6. Find a vector whose magnitude is 2 and is parallel to vector $\underline{i} + \underline{j} + \underline{k}$ (H.W) (2 times)

Sol: Let $\underline{a} = -\underline{i} + \underline{j} + \underline{k}$

Then $|\underline{a}| = \sqrt{(-1)^2 + (1)^2 + (1)^2}$

$= \sqrt{1+1+1} = \sqrt{3}$

If $\underline{u}$ is unit vector parallel to $\underline{a}$.

Then $\frac{\underline{u}}{|\underline{u}|} = \frac{\underline{a}}{|\underline{a}|}$ $\quad\quad \because |\underline{u}| = 2$

$\frac{\underline{u}}{2} = \frac{-\underline{i}+\underline{j}+\underline{k}}{\sqrt{3}}$

$\underline{u} = \frac{2}{\sqrt{3}}(-\underline{i} + \underline{j} + \underline{k})$

$\underline{u} = -\frac{2}{\sqrt{3}}\underline{i} + \frac{2}{\sqrt{3}}\underline{j} + \frac{2}{\sqrt{3}}\underline{k}$

7. Find a unit vector in the direction $\underline{V} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$. (H.W) (3 times)

Sol: Given vector is $\underline{v} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$

Then $|\underline{V}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}} = \frac{1+3}{4} = \frac{4}{4}$

$|\underline{V}| = 1$

If $\underline{u}$ is the unit vector in the direction of $\underline{v}$ then

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$\underline{u} = \frac{\underline{v}}{|\underline{v}|}$

$= \frac{\frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}}{1}$

$\underline{u} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$

8. **Find the direction cosines for $\overrightarrow{PQ}$, where P (2 , 1 , 5) , Q(1 , 3 , 1) (C.W)**

Sol: Given points are

$P = (2, 1, 5), Q = (1, 3, 1)$

Then $\overrightarrow{PQ} = (1-2)\,\underline{i} + (3-1)\,\underline{j} + (1-5)\,\underline{k}$

$\overrightarrow{PQ} = -\underline{i} + 2\underline{j} - 4\underline{k}$

$|\overrightarrow{PQ}| = \sqrt{(-1)^2 + (2)^2 + (-4)^2}$

$= \sqrt{1 + 4 + 16}$

$|\overrightarrow{PQ}| = \sqrt{21}$

Hence the direction cosines of $\overrightarrow{PQ}$ are $\left(\frac{-1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}}\right)$

9. **Find unit vector in the direction of vector $\vec{V} = 2\underline{i} - \underline{j}$. (H.W) (2 times)**

Sol: Given vector is $\underline{v} = 2\underline{i} - \underline{j}$

Then $|\underline{v}| = \sqrt{(2)^2 + (-1)^2} = \sqrt{4+1}$

$|\underline{v}| = \sqrt{5}$

If $\underline{u}$ is the unit vector in the direction of $\vec{V}$.

Then $\underline{u} = \frac{\underline{v}}{|\underline{v}|}$

$\underline{u} = \frac{2\underline{i} - \underline{j}}{\sqrt{5}}$

$= \frac{1}{\sqrt{5}}(2\underline{i} - \underline{j})$

10. **Write the vector $\overrightarrow{PQ}$ in the form $\overline{x\underline{i}} + \overline{y\underline{j}}$ where P(0 , 5) and Q (-1 , -6) (C.W)**

Sol: Given Points are P = (0 , 5) , Q = (-1 , -6)

Then $\overrightarrow{PQ} = (-1-0)\,\underline{i} + (-6-5)\,\underline{j}$

$\overrightarrow{PQ} = -\underline{i} - 11\underline{j}$

11. **Find a so that $|a\underline{i} + (a+1)\underline{j} + 2\underline{k}| = 3$ (6 times)**

Sol: Given that $|a\underline{i} + (a+1)\underline{j} + 2\underline{k}| = 3$

$\sqrt{\alpha^2 + (\alpha+1)^2 + (2)^2} = 3$

$\sqrt{\alpha^2 + \alpha^2 + 2\alpha + 1 + 4} = 3$

$\sqrt{2\alpha^2 + 2\alpha + 5} = 3$

Squaring both sides

$2\alpha^2 + 2\alpha + 5 = 9$

$2\alpha^2 + 2\alpha + 5 - 9 = 0$

$2\alpha^2 + 2\alpha - 4 = 0$

$\alpha^2 + \alpha - 2 = 0$

$\alpha^2 + 2\alpha - \alpha - 2 = 0$

$\alpha(\alpha+2) - 1(\alpha+2) = 0$

$(\alpha - 1)(\alpha + 2) = 0$

$\alpha = 1. -2$

12. **Find a vector of length 5, in the direction of opposite that of** $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$ **(C.W) (3 times)**

Sol Given

$\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$

$|\underline{v}| = \sqrt{(1)^2 + (-2)^2 + (3)^3}$

$= \sqrt{1+4+9}$

$|\underline{v}| = \sqrt{14}$

Now unit vector in the direction of $\underline{v}$

$\hat{v} = \frac{\underline{v}}{|\underline{v}|}$

$\hat{v} = \frac{\underline{i} - 2\underline{j} + 3\underline{k}}{\sqrt{14}}$

Hence required vector of length 5 in the direction opposite to the direction of $\underline{v} = -5(\hat{v})$

$$= -5\left[\frac{-5}{\sqrt{14}}\underline{i} + \frac{10}{\sqrt{14}}\underline{j} - \frac{15}{\sqrt{14}}\underline{k}\right]$$

13. **Find a unit vector in the direction of the vector** $\underline{v} = 2\underline{i} + 6\underline{j}$ **(C.W) (2 times)**

Sol Given

$\underline{v} = 2\underline{i} + 6\underline{j}$

$|\underline{v}| = \sqrt{(2)^2 + (6)^2}$

$= \sqrt{4+36}$

$= \sqrt{40}$

$= \sqrt{4 \times 10}$

$|\underline{v}| = 2\sqrt{10}$

A unit vector in the direction of vector

$\hat{v} = \frac{\underline{v}}{|\underline{v}|}$

$\hat{v} = \frac{2\underline{i} + 6\underline{j}}{2\sqrt{10}}$

$= \frac{2}{2\sqrt{10}}\underline{i} + \frac{6}{2\sqrt{10}}\underline{j}$

$\hat{v} = \frac{1}{\sqrt{10}}\underline{i} + \frac{3}{\sqrt{10}}\underline{j}$

14. **Find the direction cosines for the vector** $\underline{v} = 3\underline{i} - \underline{j} + 2\underline{k}$ **(H.W) (2 times)**

Sol Given

$\underline{v} = 3\underline{i} - \underline{j} + 2\underline{k}$

$|\underline{v}| = \sqrt{(3)^2 + (-1)^2 + (2)^2}$

$= \sqrt{9+1+4}$

$|\underline{v}| = \sqrt{14}$

Now

$$\hat{v}=\frac{\underline{v}}{|\underline{v}|}=\frac{3\underline{i}-\underline{j}+2\underline{k}}{\sqrt{14}}$$

$$\hat{v}=\frac{3}{\sqrt{14}}\underline{i}-\frac{1}{\sqrt{14}}\underline{j}+\frac{2}{\sqrt{14}}\underline{k}$$

Direction cosines of $\underline{v}$ are

$$\left(\frac{3}{\sqrt{14}},\frac{-1}{\sqrt{14}},\frac{2}{14}\right)$$

Topic II: Scalar Product of Vector:

15. Find a vector whose magnitude is 4 and is parallel to $2\underline{i}-3\underline{j}+6\underline{k}$

(H.W) (5 times)

Sol: Let $\underline{v}=2\underline{i}-3\underline{j}+6\underline{k}$

We further suppose that $\underline{u}$ is a vector whose magnitude is 4 and is parallel to $\underline{v}$. As unit vector of $\underline{u}=\frac{\underline{v}}{|\underline{v}|}$

Also $|\underline{v}|=\sqrt{(2)^2(-3)^2+(6)^2}=\sqrt{4+9+36}=\sqrt{49}=7$

Therefore, unit vector of $\underline{u}=\frac{2\underline{i}-3\underline{j}+6\underline{k}}{7}=\frac{2}{7}\underline{i}-\frac{3}{7}\underline{j}+\frac{6}{7}\underline{k}$

Therefore, $\underline{u}=|\underline{u}|\times$ unit vector of $\underline{u}$

$$=4\left(\frac{2}{7}\underline{i}-\frac{3}{7}\underline{j}+\frac{6}{7}\underline{k}\right)=\frac{8}{7}\underline{i}-\frac{12}{7}\underline{j}+\frac{24}{7}\underline{k}$$

16. Find $\propto$ so that vectors $\underline{u}=\propto\underline{i}+\underline{2}\propto\underline{j}-\underline{k}$, $\underline{v}=\underline{i}+\propto\underline{j}+3\underline{k}$ are perpendicular.

(H.W) (4 times)

Sol: Given vectors are $\underline{u}=\propto\underline{i}+2\propto\underline{j}-\underline{k}$ and $\underline{v}=\underline{i}+\propto\underline{j}+3\underline{k}$

Then $\underline{u}.\underline{v}=(\propto\underline{i}+2\propto\underline{j}-\underline{k}).(\underline{i}+\propto\underline{j}+3\underline{k})$

$=\propto(1)+2\propto(\propto)+(-1)(3)$

$=\propto+2\propto^2-3$

$u.v=2\propto^2+\propto-3$

As $\underline{u}$ & $\underline{v}$ are perpendicular

So. $\underline{u}.\underline{v}=0$

$2\propto^2+\propto-3=0$

$2\propto^2+3\propto-2\propto-3=0$

$\propto(2\propto+3)-1(2\propto+3)=0$

$(2\propto+3)(\propto-1)=0$

$\propto=-{}^3/_2, 1$

17. Show that vectors $\underline{u}=\underline{i}+2\underline{j}-\underline{k}$ and $\underline{v}=-\underline{i}+\underline{j}+\underline{k}$ are perpendicular to each other.

(C.W) (2 times)

Sol Let

$\underline{u}=\underline{i}+2\underline{j}-\underline{k}$, $\underline{v}=-\underline{i}+\underline{j}+\underline{k}$

Are perpendicular to each other

If $\underline{u}.\underline{v}=0$

Now

$\underline{u}.\underline{v} = (\underline{i}+2\underline{j}-\underline{k}).(-\underline{i}+\underline{j}+\underline{k})$
$= 1(-1) + 2(1) + (-1)(1)$
$= -1 + 2 - 1$
$= -2 + 2$
$= 0$
Hence $\underline{u}.\underline{v} = 0$ so $\underline{u}$ and $\underline{v}$ are perpendicular to each other

18. If $\underline{v}$ is a vector for which $\underline{v}.\underline{i} = 0$ $\underline{v}.\underline{k} = 0$ and find $\underline{v}$ (C.W) (2 times)

Sol Let
$\underline{v} = x\underline{i} + y\underline{j} + z\underline{k}$
As $\underline{v}.\underline{i} = 0$
$(x\underline{i} + y\underline{j} + z\underline{k}).(\underline{i} + u\underline{j} + o\underline{k}) = 0$
$\Rightarrow x(1) + y(0) + z(0) = 0$
$\Rightarrow x = 0$
As $\underline{v}.\underline{j} = 0$
$(x\underline{i} + y\underline{j} + z\underline{k}).(0\underline{i} + \underline{j} + 0\underline{k}) = 0$
$\Rightarrow x(0) + y(1) + z(0) = 0$
$\Rightarrow y = 0$
As $\underline{v}.\underline{k} = 0$
$(x\underline{i} + y\underline{j} + z\underline{k}).(0\underline{i} + 0\underline{j} + \underline{k}) = 0$
$\Rightarrow x(0) + y(0) + z(1) = 0$
$\Rightarrow z = 0$
Hence $\underline{v} = x\underline{i} + y\underline{j} + z\underline{k}$
$\underline{v} = 0\underline{i} + 0\underline{j} + 0\underline{k}$
$\underline{v} = \underline{0}$ **(Null vector)**

Topic III: Vector Product:

19. Prove that $\underline{a}\times(\underline{b}+\underline{c})+\underline{b}\times(\underline{c}+\underline{a})+\underline{c}\times(\underline{a}+\underline{b})=\underline{0}$ (3 times)

Sol: L.H.S $= \underline{a}\times(\underline{b}+\underline{c})+\underline{b}\times(\underline{c}+\underline{a})+\underline{c}\times(\underline{a}+\underline{b})$
$(\underline{a}\times\underline{b})+(\underline{a}\times\underline{c})+(\underline{b}\times\underline{c})+(\underline{b}\times\underline{a})+(\underline{c}\times\underline{a})+(\underline{c}\times\underline{b})$
We know that $\underline{a}\times\underline{b} \neq \underline{b}\times\underline{a}$
But $\underline{a}\times\underline{b} = -(\underline{b}\times\underline{a})$
$(\underline{a}\times\underline{b})+(\underline{a}\times\underline{c})+(\underline{b}\times\underline{c})-(\underline{a}\times\underline{b})-(\underline{a}\times\underline{c})-(\underline{b}\times\underline{c}) = 0 =$ R. H.S

20. If $\underline{a}+\underline{b}+\underline{c}=0$, then prove that $\underline{a}\times\underline{b}=\underline{b}\times\underline{c}=\underline{c}\times\underline{a}$ (C.W) (5 times)

Sol Given
Since $\underline{a}+\underline{b}+\underline{c}=0$
Taking cross – product with $\underline{a}$
$\underline{a}\times(\underline{a}+\underline{b}+\underline{c})=\underline{a}\times 0$
$\underline{a}\times\underline{a}+\underline{a}\times\underline{b}+\underline{a}\times\underline{c}=0$
$\because \underline{a}\times\underline{a}=0$
$0+\underline{a}\times\underline{b}+\underline{a}\times\underline{c}=0$
$\Rightarrow \underline{a}\times\underline{b}=-\underline{a}\times\underline{c}$
$\because -\underline{a}\times\underline{c}=\underline{c}\times\underline{a}$

$\underline{a}\times\underline{b}=\underline{c}\times\underline{a}$ ________ (1)

Also taking cross – product with $\underline{b}$

$\underline{b}\times(\underline{a}+\underline{b}+\underline{c})=\underline{b}\times 0$

$\underline{b}\times\underline{a}+\underline{b}\times\underline{b}+\underline{b}\times\underline{c}=0$

$\because \underline{b}\times\underline{b}=0$

$\underline{b}\times\underline{a}+0+\underline{b}\times\underline{c}=0$

$\underline{b}\times\underline{c}=-\underline{b}\times\underline{a}$

$\because -\underline{b}\times\underline{a}=\underline{a}\times\underline{b}$

$\underline{b}\times\underline{c}=\underline{a}\times\underline{b}$ __________ (2)

From (1) and (2)

$\underline{a}\times\underline{b}=\underline{b}\times\underline{c}=\underline{c}\times\underline{a}$ **Ans.**

Topic IV: Application of Vector:

21. Prove that the vectors $i-2j+3k$, $-2i+3j-4k$ and $i-3j+5k$ are coplanar.

(H.W) (3 times)

Sol: vectors are coplanar

If $\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}=0$

Now,

$$\text{L.H.S}=\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}=0$$

$=1(15-12)+2(-10+4)+3(6-3)$

$=3-12+9=12-2=0$

Hence det = 0, So vectrors are coplanar.

22. A force F= 7i+4j-3k is applied at P (1,-2,3). Find its moment about the the point Q (2,1,1). **(C.W)**

Sol: here F= 7i+4j-3k

$\vec{r}=\overline{QP}$

$=(1-2)i+(-2-1)j+(3-1)k$

$\vec{r}=-\underline{i}-3\underline{j}+2\underline{k}$

Moment $=\vec{r}\times\vec{f}$

$$\begin{vmatrix} i & j & k \\ -1 & -3 & 2 \\ 7 & 4 & -3 \end{vmatrix}$$

$=\underline{i}(9-8)-j(3-14)+\underline{k}(-4+21)$

$= \underline{i} + 11\underline{j} + 17\underline{k}$

23. Find α if $\underline{i} - \underline{j} + \underline{k}$, $i - 2\underline{j} - 3\underline{k}$ and $3\underline{i} - a\underline{j} + 5\,\underline{k}$ are coplanar. (C.W) (5 times)

Sol: Suppose $\underline{u} = \underline{i} - \underline{j} + \underline{k}$

$\underline{v} = \underline{i} - 2\underline{j} - 3\underline{k}$

$\underline{w} = 3\underline{i} - \alpha\,\underline{j} - 5\underline{k}$

As $\underline{u}$, $\underline{v}$ & $\underline{w}$ are coplanar

So

$$\begin{vmatrix} 1 & -1 & 1 \\ 1 & -2 & -3 \\ 3 & -\alpha & 5 \end{vmatrix} = 0$$

Expanding from R_1

$1(-10 - 3\alpha) + 1(5 + 9) + 1(-\alpha + 6) = 0$

$-10 - 3\alpha + 14 - \alpha + 6 = 0$

$-4\alpha + 10 = 0$

$\alpha = \frac{10}{4} = \frac{5}{2}$

24. A force $\underline{F} = 4\underline{i} - 3\underline{k}$ passes through the point A(2 , -2 , 5). Find moment of $\underline{F}$ about the point B(1 , -3 , 1). (H.W)

Sol: Given force is $\underline{F} = 4\underline{i} - 3\underline{k}$

Point of application = A (2, -2 , 5)

Point of rotation = B(1 , -3 , 1)

Then $\underline{r} = \overrightarrow{BA}$

$\underline{r} = (2 - 1)\,\underline{i} + (-2 + 3)\,\underline{j} + (5 - 1)\,\underline{k}$

$\underline{r} = \underline{i} + \underline{j} + 4\underline{k}$

Then moment of $\vec{F}$ about B is

$\underline{M} = \underline{r} \times \underline{F}$

$$= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & 4 \\ 4 & 0 & -3 \end{vmatrix}$$

$= (-3 - 0)\,\underline{i} - (-3 - 16)\,j + (0 - 4)\,\underline{k}$

$\underline{M} = -3\underline{i} + 19\underline{j} - 4\underline{k}$

25. Prove that vectors $i - 2j + 3k$, $-2i + 3j - 4k$ and $i - 3j + 5k$ are coplaner. (C.W) (5 times)

Sol: Given vectors are $\underline{u} = \underline{i} - 2\underline{j} + 3\underline{k}$, $\underline{v} = -2\underline{j} + 3\underline{j} - 4\underline{k}$ and $\underline{w} = \underline{i} - 3\underline{j} + 5\underline{k}$

If $\underline{u}$, $\underline{v}$ & $\underline{w}$ are coplanar

then

$$\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

$\Rightarrow 1(15 - 12) + 2(-10 + 4) + 3(6 - 3) = 0$

$\Rightarrow 3 + 2(-6) + 3(3) = 0$

$\Rightarrow 3 - 12 + 9 = 0$

$\Rightarrow 12 - 12 = 0 \quad \Rightarrow \quad 0 = 0$

Hence $\underline{u}$, $\underline{v}$ & $\underline{w}$ are coplanar

26. **Find the value of 'α', so that the vectors $\alpha\underline{i}+\underline{j}$, $\underline{i}+\underline{j}+3\underline{k}$ and $2\underline{i}+\underline{j}-2\underline{k}$ are coplanar.** **(C.W) (4 times)**

Sol Let
$\underline{u}=\alpha\underline{i}+\underline{j}+0\underline{k}$
$\underline{v}=\underline{i}+\underline{j}+3\underline{k}$
$\underline{w}=2\underline{i}+\underline{j}-2\underline{k}$
As vectors are coplanar.
If

$$\begin{vmatrix}\alpha & 1 & 0\\ 1 & 1 & 3\\ 2 & 1 & -2\end{vmatrix}=0$$

$$\alpha(-2-3)-1(-2-6)+0(1-2)=0$$
$$-5\alpha+8+0=0$$
$$-5\alpha+8=0$$
$$8=5\alpha$$
$$\alpha=\frac{8}{5}\qquad \text{Ans.}$$

27. **Find a vector perpendicular to each of the vectors $\underline{a}=2\underline{i}-\underline{j}+\underline{k}$ and $\underline{b}=4\underline{i}+2\underline{j}-\underline{k}$** **(C.W) (2 times)**

Sol: Let $\underline{a}=2\underline{i}+\underline{j}+\underline{k}$
$\underline{b}=4\underline{i}+2\underline{j}-\underline{k}$

Now

$$\underline{a}\times\underline{b}=\begin{vmatrix}i & j & k\\ 2 & -1 & 1\\ 4 & 2 & -1\end{vmatrix}$$

$$\underline{a}\times\underline{b}=\underline{i}(1-2)-\underline{j}(-2-4)+\underline{k}(4+4)$$
$$=-\underline{i}+6\underline{j}+8\underline{k}$$

Now Verification:

$$\underline{a}\cdot\underline{a}\times\underline{b}=(2\underline{i}-\underline{j}+\underline{k}).(-\underline{i}+6\underline{j}+8\underline{k})$$
$$=2(-1)+(-1)(6)+(1)8$$
$$=-2-6+6$$
$$=0$$

And $\underline{b}\cdot\underline{a}\times\underline{b}=(4\underline{i}+2\underline{j}-\underline{k}).(-\underline{i}+6\underline{j}+8\underline{k})$

$$=4(-1)+2(6)+(-1)8$$
$$=-4+12-8$$
$$=-12+12$$
$$=0$$

Hence $\underline{a}\times\underline{b}$ is perpendicular to both the vectors $\underline{a}$ and $\underline{b}$

28. **Find the direction cosines of $\underline{v}=6\underline{i}-2\underline{j}+\underline{k}$** **(H.W) (2 times)**

Sol: $\underline{v}=6\underline{i}-2\underline{j}+\underline{k}$

Now $|\underline{v}|=\sqrt{(6)^2+(-2)^2+(1)^2}$

$$|\underline{v}| = \sqrt{36+4+1} = \sqrt{41}$$

We know that

$$\hat{v} = \frac{\underline{v}}{|\underline{v}|} = \frac{6\underline{i} - 2\underline{j} + \underline{k}}{\sqrt{41}}$$

$$\hat{v} = \frac{6}{\sqrt{41}}\underline{i} - \frac{2}{\sqrt{41}}\underline{j} + \frac{1}{\sqrt{41}}\underline{k}$$

Hence Direction cosines of $\underline{v}$ are

$$\left(\frac{6}{\sqrt{41}}, \frac{-2}{\sqrt{41}}, \frac{1}{\sqrt{41}}\right)$$

29. Find magnitude of the vector $\underline{v}$ and write the direction cosines of $\underline{v}$ where **(H.W)**

$\underline{v} = 2\underline{i} + 3\underline{j} + 4\underline{k}$

Sol: Given $\underline{v} = 2\underline{i} + 3\underline{j} + 4\underline{k}$

$$|\underline{v}| = \sqrt{(2)^2 + (3)^2 + (4)^2}$$

$$= \sqrt{4+9+16}$$

$$|\underline{v}| = \sqrt{29}$$

We know that

$$\hat{v} = \frac{\underline{v}}{|\underline{v}|}$$

$$= \frac{2\underline{i} + 3\underline{j} + 4\underline{k}}{\sqrt{29}}$$

$$= \frac{2}{\sqrt{29}}\underline{i} + \frac{3}{\sqrt{29}}\underline{j} + \frac{4}{\sqrt{29}}\underline{k}$$

Hence direction cosines of $\underline{v}$ are

$$\left(\frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}\right)$$

30. If $\underline{v}$ is a vector for which $\underline{v}.\underline{i} = 0$, $\underline{v}.\underline{j} = 0$, $\underline{v}.\underline{k} = 0$ find $\underline{v}$ **(C.W)**

Sol: Let $\underline{v} = x\underline{i} + y\underline{j} + z\underline{k} \rightarrow$ (i)

Given $\underline{v}.\underline{i} = 0$

$$\Rightarrow (x\underline{i} + y\underline{j} + z\underline{k}).\underline{i} = 0$$

$$\Rightarrow (x\underline{i} + y\underline{j} + z\underline{k}).(\underline{i} + 0\underline{j} + 0\underline{k}) = 0$$

$$\Rightarrow (x)(1) + (y)(0) + z(0) = 0$$

$$\Rightarrow x + 0 + 0 = 0$$

$$\Rightarrow x = 0$$

Similarly from $\underline{v}.\underline{j} = 0$ *and* $\underline{v}.\underline{k} = 0$

$\Rightarrow y = 0 \qquad \Rightarrow z = 0$

putting in (i) we get

$\underline{v} = 0\underline{i} + 0\underline{j} + 0\underline{k} = \underline{0}$ (Null Vector)

31. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ **(C.W)**

Sol: Given $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

Taking cross product with $\vec{a}$.

$\Rightarrow \vec{a}\times\left(\vec{a}+\vec{b}+\vec{c}\right)=\vec{a}\times\vec{0}$

$\vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=0$

$0+\vec{a}\times\vec{b}-\vec{c}\times\vec{a}=0$

$\vec{a}\times\vec{b}=\vec{c}\times\vec{a}=0 \quad \rightarrow$ (i)

also $\quad \vec{a}+\vec{b}+\vec{c}=0$

taking cross product with $\vec{b}$

$\vec{b}\times\left(\vec{a}+\vec{b}+\vec{c}\right)=\vec{b}\times\vec{o}$

$\vec{b}\times\vec{a}+\vec{b}\times\vec{b}+\vec{b}\times\vec{c}=0$

$\vec{b}\times\vec{a}+0+\vec{b}\times\vec{c}=0$

$-\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=0$

$\Rightarrow \vec{a}\times\vec{b}=\vec{b}\times\vec{c} \quad \rightarrow$ (i)

From (i) and (ii) we get

$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Hence proved.

32. Find a unit vector in the direction of $\vec{v}=\vec{i}+2\vec{j}-\vec{k}$ **(H.W)**

Sol: Then $\left|\underline{V}\right|=\left|\underline{i}+2\underline{j}-\underline{k}\right|$

$$\left|\underline{V}\right|=\sqrt{(1)^2+(+2)^2+(-1)^2}$$

$$\left|\underline{V}\right|=\sqrt{1+4+1}=\sqrt{6}$$

Now

Unit vector in the direction of given vector $V=\dfrac{-\underline{i}+2\underline{j}-\underline{k}}{\sqrt{6}}$

33. Find the vector from point A to the origion where $\overline{AB}=4\underline{i}-2\underline{j}$ and B is the point $(-2,5)$ **(C.W)**

Sol: $\because \overline{AB}=\overline{OB}-\overline{OA}$

$$\Rightarrow 4\underline{i}-2\underline{j}=-2\underline{i}+5\underline{j}-\overline{OA}$$

or

$$\overline{OA}=-2\underline{i}+5\underline{j}-4\underline{i}+2\underline{j}$$

$$\overline{OA}=-6\underline{i}+7\underline{j}$$

Now for required vector from A to origin.

so

$$-\overline{AO}=-6\underline{i}+7\underline{j}$$

$$\Rightarrow \overline{AO}=6\underline{i}-7\underline{j}$$

LONG QUESTIONS OF CHAPTER-7 ACCORDING TO ALP SMART SYLLABUS-2020

Topic I: Vector in Space:

1. Find a vector of length 5 in the direction opposite that of $v = \underline{i} - 2\underline{j} + 3\underline{k}$ **(H.W)**
2. Find a vector whose magnitude is 4 and parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$ **(H.W)**
3. Find the vector from the point A to the origin where $\overline{AB} = 4\hat{i} - 2\hat{j}$ and B is the point (-2, 5) **(C.W)**

Topic II: Scalar Product of Vector:

4. Prove by vector method that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ **(C.W)** **(2 tiemes)**
5. Prove that in triangle ABC, $c^2 = a^2 + b^2 - 2ab\cos C$ **(C.W)**
6. Prove that in any triangle ABC by vector method $a^2 = b^2 + c^2 - 2bc\cos A$. **(C.W)**

Topic III: Vector Product:

7. Prove that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ by vector method. **(H.W) (4 times)**
8. If $\underline{a} + \underline{b} + \underline{c} = 0$, then prove that $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$ **(C.W) (4 times)**

Topic IV: Application of Vector:

9. Prove that the vectors $\underline{i} - 2\underline{j} + 3\underline{k}, -2\underline{i} + 3\underline{j} - 4\underline{k}$ and $\underline{i} - 3\underline{j} + 5\underline{k}$ are coplanar. **(H.W) (2 times)**
10. A force of magnitude '6' units acting parallel to the $2\underline{i} - 2\underline{j} + \underline{k}$ displaces, the point of application from (1, 2, 3) to (5, 3, 7). Find the work done. **(H.W)**
11. Find volume of tetrahedron with the vertices (0, 1, 2), (3, 2, 1), (1, 2, 1) and (5, 5, 6). **(H.W) (3 times)**
12. Prove that in any triangle ABC $b^2 = c^2 + a^2 - 2ca\cos\beta$ **(H.W)**
13. Find the value of α so that $\alpha i + j$, $i + j + 3k$ and $2i + j - 2k$ are coplanar.
14. Find the constant a such that the vectors are coplanar $\vec{i} - \vec{j} + \vec{k}$, $\vec{i} - 2\vec{j} - 3\vec{k}$ and $3\vec{i} - a\vec{j} + 5\vec{k}$. **(H.W)**
15. Find volume of the tetrahedron whose vertices are A(2,1,8), B (3, 2, 9), C (2, 1, 4), D (3, 3, 10) **(H.W) (5 times)**

Chapter-7 (Examples According to ALP Smart Syllabus)

Example 1: (Page#361) Find the volume of the parallelepiped determined by

$\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$, $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$, $\underline{w} = \underline{i} - 7\underline{j} - 4\underline{k}$

Sol: Volume of the parallelepiped $= \underline{u}.\underline{v} \times \underline{w} = \begin{vmatrix} 1 & 2 & -1 \\ 1 & -2 & 3 \\ 1 & -7 & -4 \end{vmatrix}$

Volume $= 1(8 + 21) - 2(-4 - 3) - 1(7 + 2) = 29 + 14 + 5 = 48$